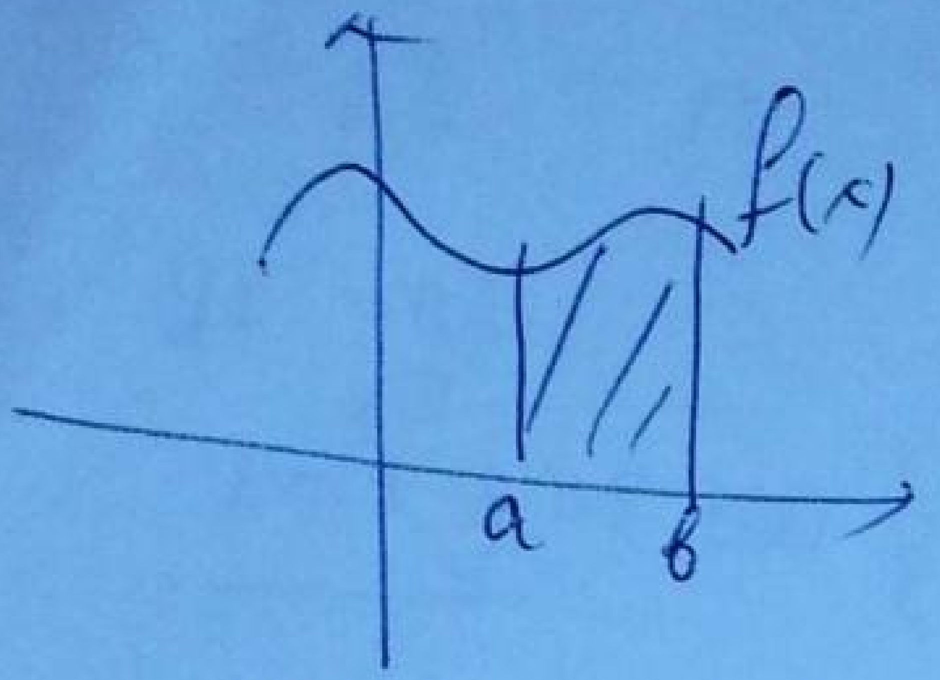
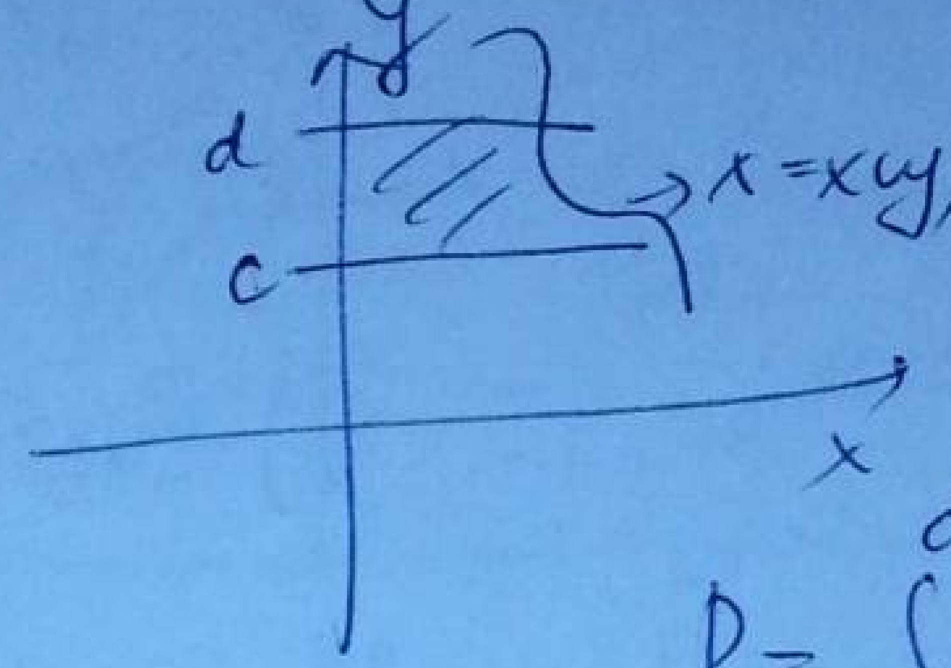
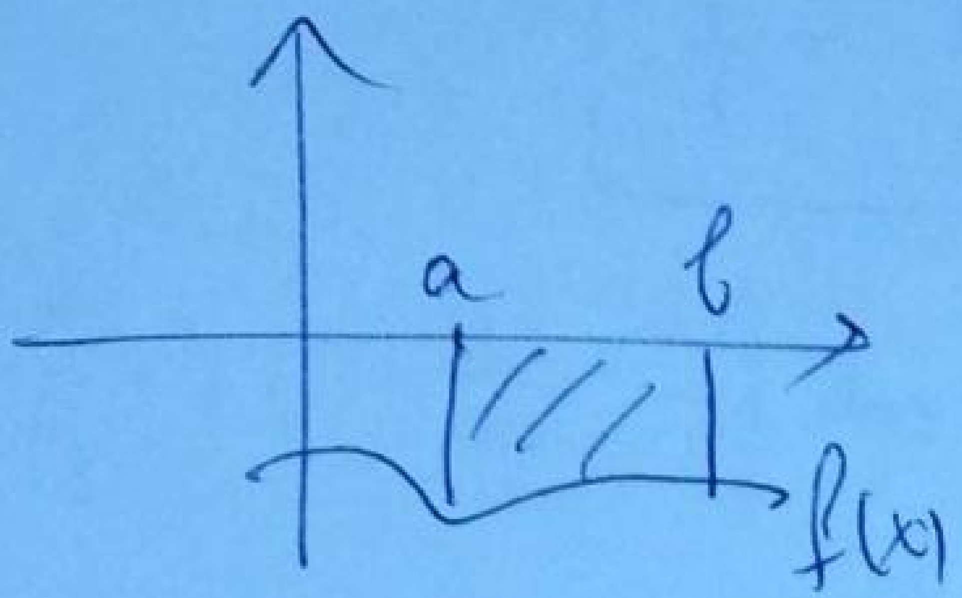
Određeni integral



$$P=\int_a^b f(x)\,dx$$



$$P=\int_c^d x(y)\,dy.$$



$$P=-\int_a^b f(x)\,dx.$$

Njutn-Lajbnicova f.

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b)-F(a)$$

1) Izračunati određene integrale:

a) $$\int_1^3 (x^3+4x+e^x)\,dx = \left[\frac{x^4}{4}+4\frac{x^2}{2}+e^x\right]\Big|_1^3 = \left(\frac{3^4}{4}+2\cdot 3^2+e^3\right) - \left(\frac{1}{4}+1+e\right).$$

b) $$\int_0^2 x e^{4x^2+1}\,dx = \left\lceil \begin{array}{l} 4x^2+1=t \\ 8xdx=dt \\ xdx=\frac{dt}{8} \end{array} \quad \begin{array}{c|c|c} x & 0 & 2 \\ \hline t & 1 & 17 \end{array} \right\rfloor =$$

$$=\frac{1}{8}\int_1^{17} e^t\,dt = \frac{1}{8}e^t\Big|_1^{17} = \underline{\frac{1}{8}(e^{17}-e)}$$

c) $$\int_0^{\pi/2} x^2 \sin(3x)\,dx = \left\lceil \begin{array}{ll} x^2=u & v=-\frac{1}{3}\cos 3x \\ 2xdx=du & \end{array} \right\rfloor =$$

$$= x^2\cdot\frac{-1}{3}\cos 3x\Big|_0^{\pi/2} + \frac{2}{3}\int_0^{\pi/2} x\cos 3x\,dx \quad -\left(-\frac{1}{3}\frac{\pi^2}{4}\cos\frac{3\pi}{2} - 0\right)+\frac{2}{3}I_1$$

$$I_1 = \int_0^{\pi/2} x\cos 3x\,dx = \left\lceil \begin{matrix} x = u & \cos 3x\,dx = dv \\ dx = du & v = \frac{1}{3}\sin 3x \end{matrix} \right\rfloor =$$

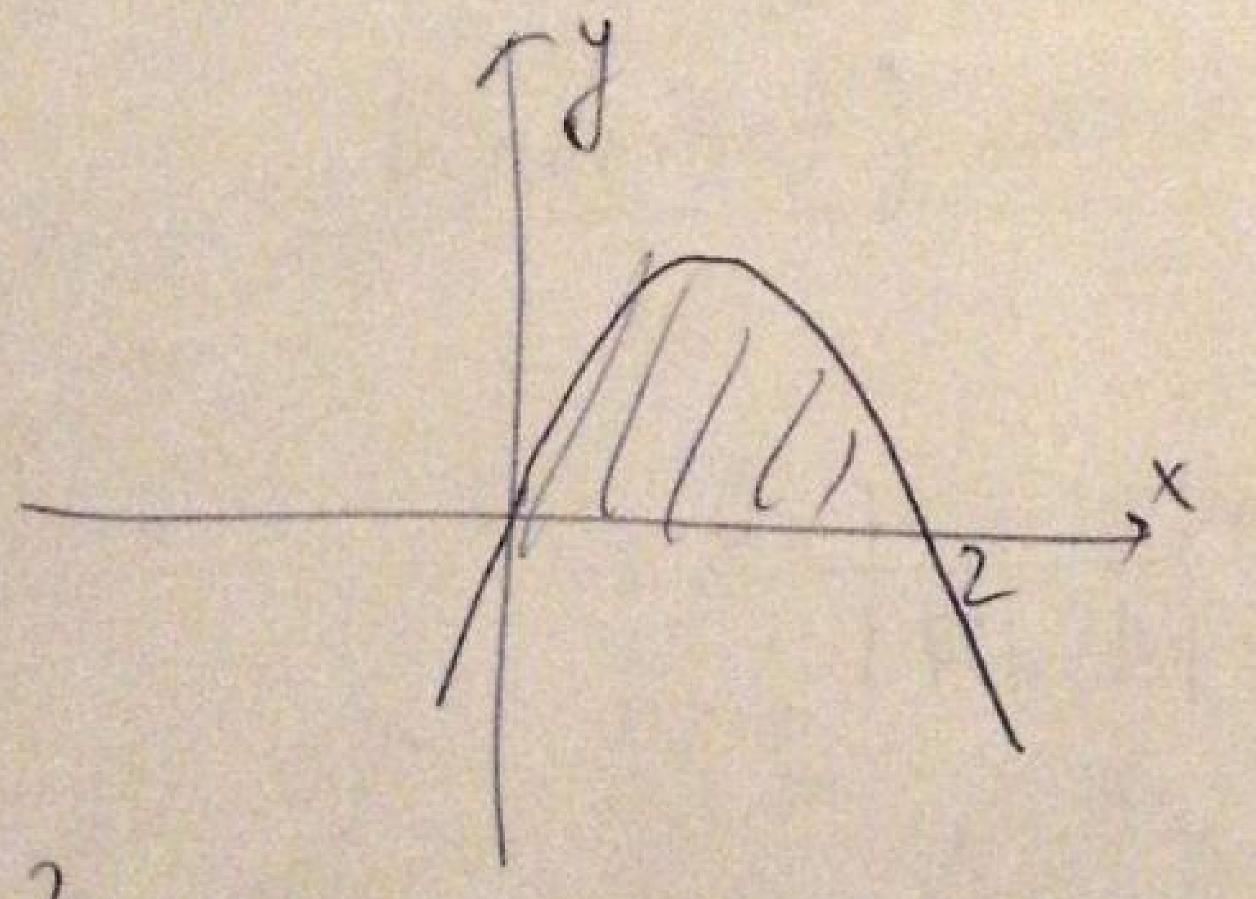
$$= x\,\frac{1}{3}\sin 3x\Big|_0^{\pi/2} - \frac{1}{3}\int_0^{\pi/2}\sin 3x\,dx = \frac{\pi}{6}\sin\frac{3\pi}{2} + \frac{1}{3}\,\frac{1}{3}\cos 3x\Big|_0^{\pi/2} =$$

$$= \frac{\pi}{6}(-1) + \left[\underbrace{\frac{1}{9}\cos\frac{3\pi}{2}}_{=0} - \frac{1}{9}\underbrace{\cos 0}_{=1}\right] = \boxed{\frac{-\pi}{6} - \frac{1}{9}}$$

d) $\int_1^2 \sqrt[3]{x^5}\,dx$, e) $\int_0^m \frac{dx}{3x+1}$, f) $\int_0^{\sqrt{7}} x^2 e^{2x}\,dx$.

2) Izračunati površinu figure ograničene linijom krive $y = -x^2 + 2x$ i pravom $y = 0$

Rj



$y = -x^2 + 2x = 0$

$x(2-x) = 0$

$x = 0 \vee x = 2$

$$P = \int_0^2 (-x^2 + 2x)\,dx = \left(-\frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}}\right)\Big|_0^2 = \frac{-8}{3} + 4 = \frac{4}{3}$$

3) Izračunati P figure ograničene sa $7x^2 - 9y + 9 = 0$ i $5x^2 - 9y + 27 = 0$

Rj $\quad -9y = -7x^2 - 9 \qquad\qquad 9y = 5x^2 + 27$

$\quad \underline{y = \frac{7}{9}x^2 + 1} \qquad\qquad \underline{y = \frac{5}{9}x^2 + 3}$

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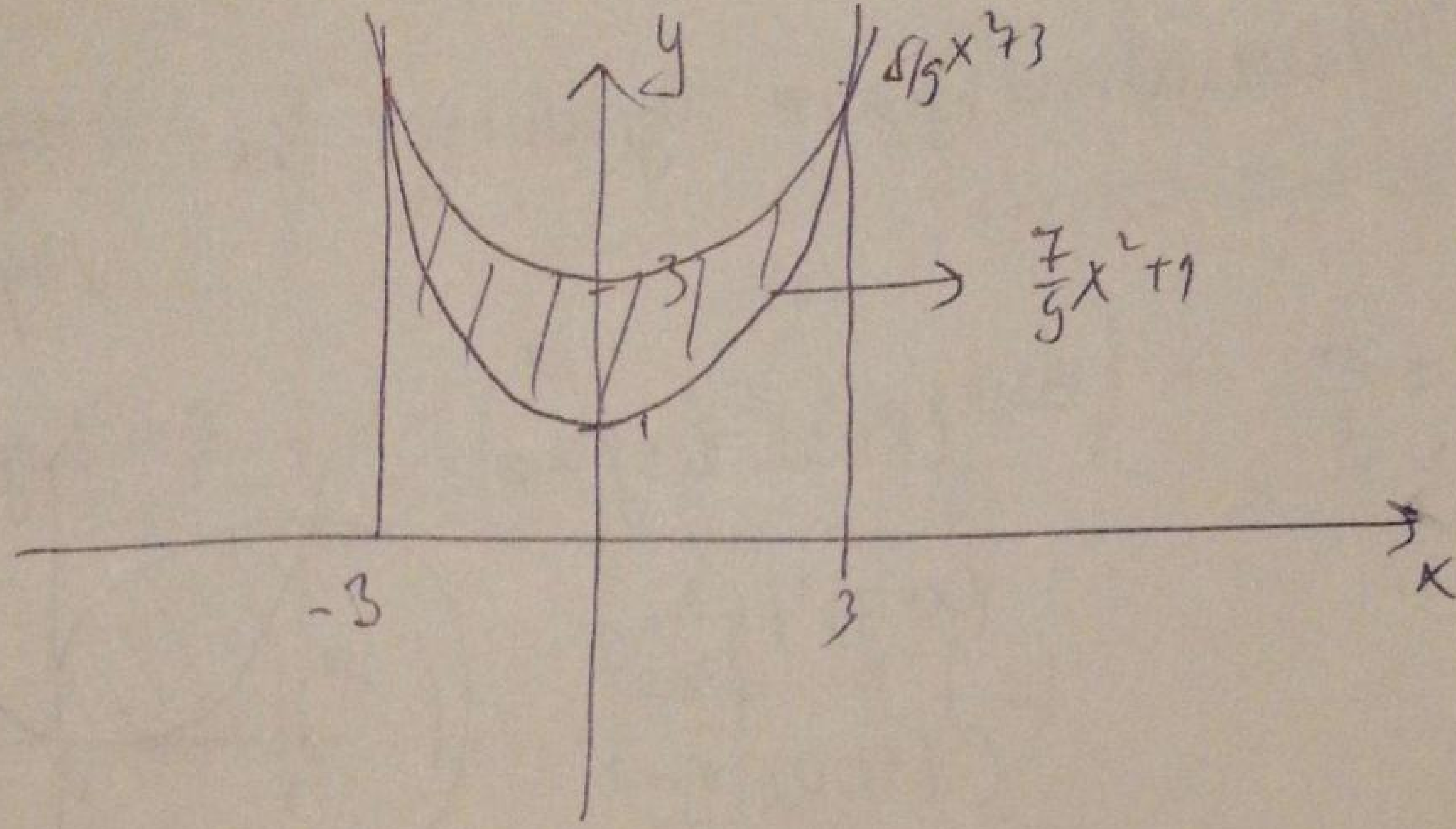
$$\frac{7}{9}x^2+1=\frac{5}{9}x^2+3 \quad /\cdot 9$$

$$7x^2-5x^2=18$$

$$2x^2=18$$

$$\underline{x=\pm 3}$$



$$P=\int_{-3}^{3}\left(\frac{5}{9}x^2+3-\frac{7}{9}x^2-1\right)dx=\ldots=8.$$

4) Izračunati P figure ograničene sa $y=-x^2+10x-16$ i $y=x+2$.

Rj.

$$y=-x^2+10x-16=0$$

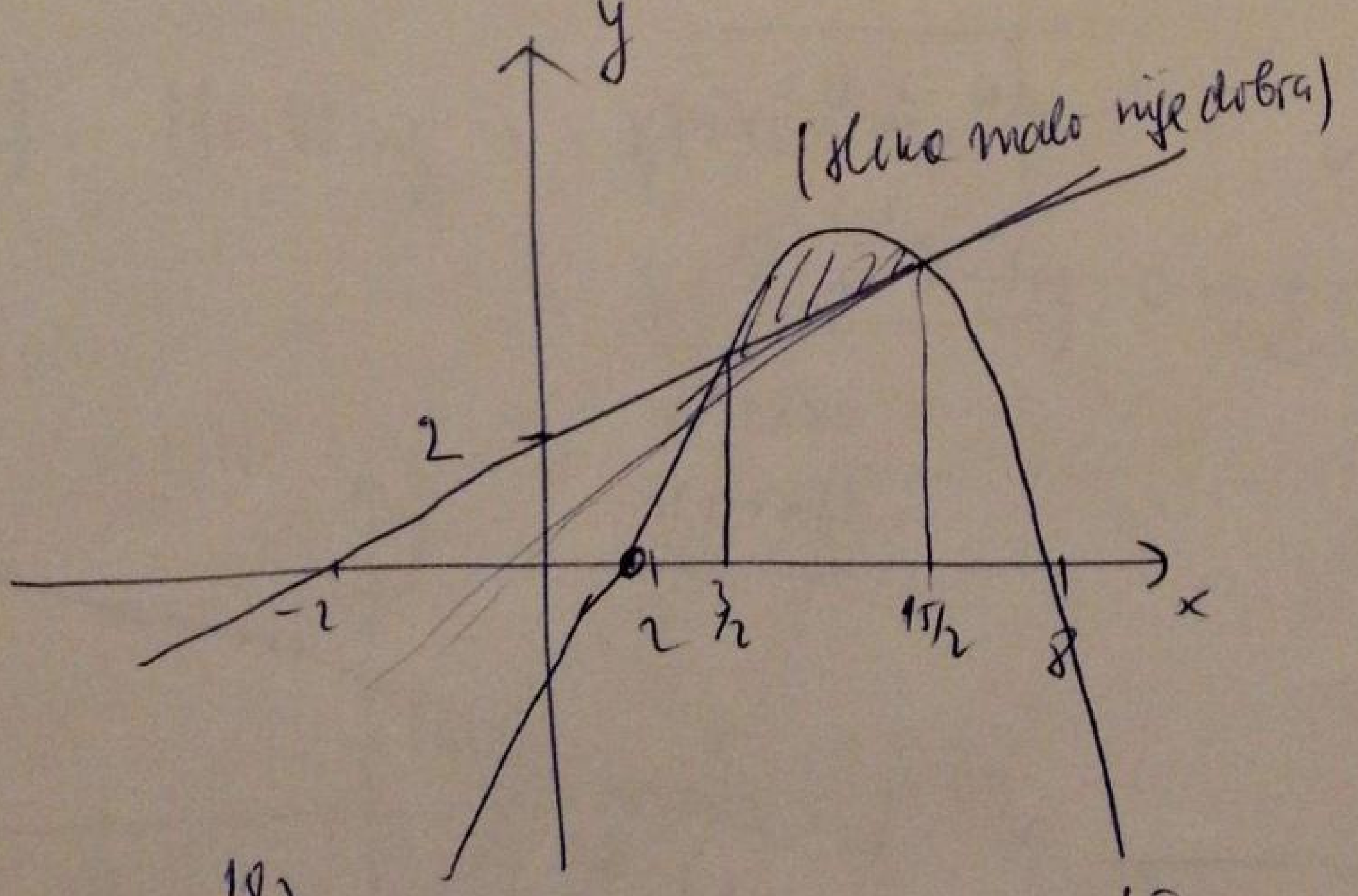
$$x_{1,2}=\frac{-10\pm\sqrt{100-64}}{-2}=\frac{-10\pm 6}{-2} \to x_1=2, \quad x_2=8$$

$$x+2=-x^2+10x-10$$

$$x^2-9x+12=0$$

$$x_{1,2}=\frac{9\pm\sqrt{81-44}}{2}=\frac{9\pm 6}{2}$$

$$x_1=\frac{15}{2}, \quad x_2=\frac{3}{2}$$



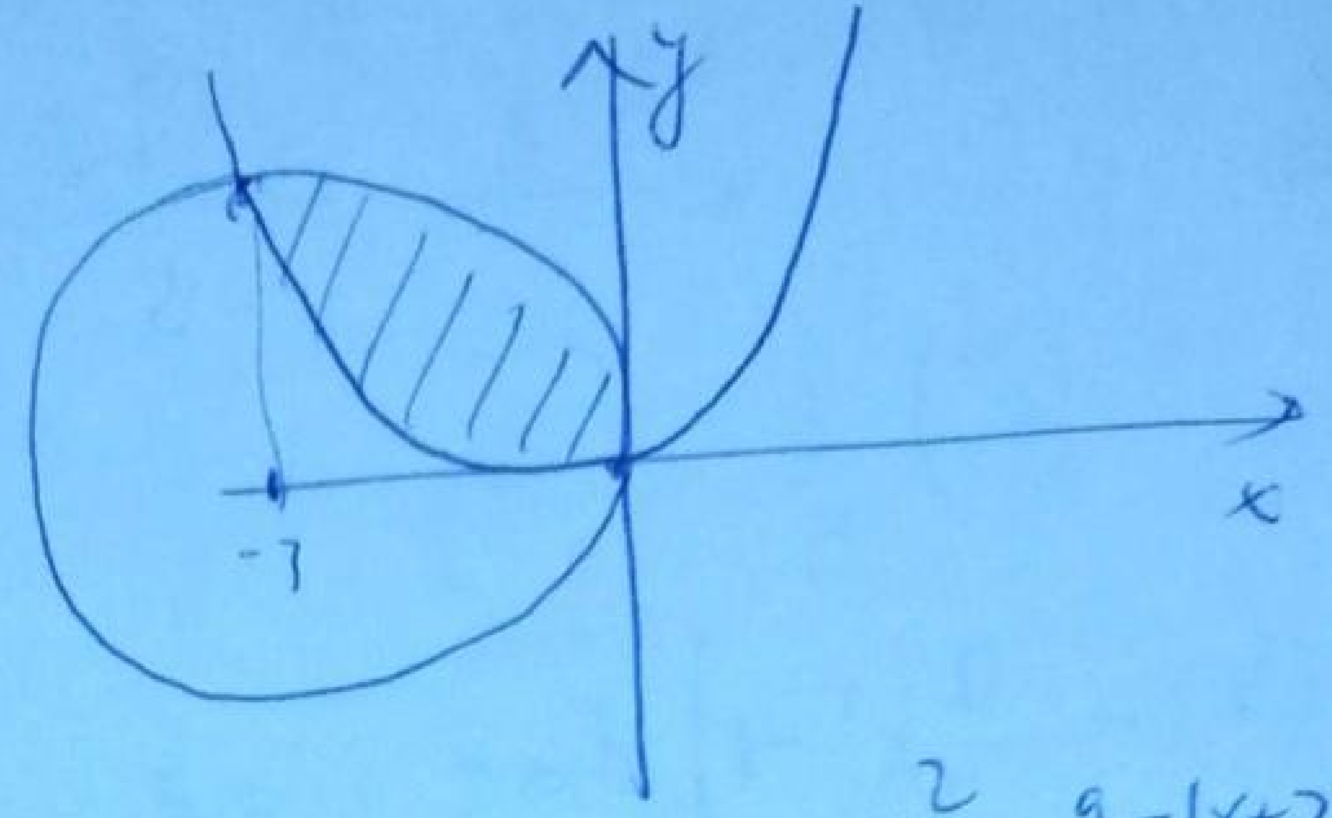
$$P=\int_{3/2}^{15/2}\left((-x^2+10x-16)-(x+2)\right)dx=\int_{3/2}^{15/2}(-x^2+9x-12)\,dx=\left[\frac{-x^3}{3}+\frac{9x^2}{2}-12x\right]\Bigg|_{3/2}^{15/2}=\frac{9}{2}$$

5) Izračunati P figure ograničene sa $3y=x^2$ i
$x^2+y^2+6x=0$

Rj: $y=\frac{x^2}{3}$ i $(x+3)^2-9+y^2=0$

$(x+3)^2+y^2=9$

$C(-3,0),\ r=3$



$y^2=9-(x+3)^2$

$y=\pm\sqrt{9-(x+3)^2}$

Presjek: $\cancel{3y+y^2+}$ $x^2+\left(\frac{x^2}{3}\right)^2+6x=0$

$x^2+\frac{x^4}{9}+6x=0$

$x\left(\frac{x^3}{9}+x+6\right)=0$

$x=0 \quad \vee \quad \frac{x^3}{9}+x+6=0 \,/\cdot 9$

$x^3+9x+54=0$

$p(-3)=-27+27+54=0$

$(x^3+9x+54):(x+3)=x^2-3x+18$

$-x^3 \pm 3x^2$

$-3x^2+9x+54$

$-3x^2 \mp 9x$

$18x+54$

$18x+54$

0

$(x+3)(x^2-3x+18)=0$

V_0

$x=-3$

$$P=\int_{-3}^{0}\left(\sqrt{9-(x+3)^2}-\frac{x^2}{3}\right)dx=\int_{-3}^{0}\sqrt{9-(x+3)^2}\,dx-\frac{1}{3}\frac{x^3}{3}\Big|_{-3}^{0}=I_1-\frac{1}{9}(0+27)$$

$I_1=$ $x+3=3\sin t$, $dx=3\cos t\,dt$

x	-3	0
t	0	$\pi/2$

$I_1 = 3\int_0^{\pi/2} \sqrt{9-9\sin^2 t}\cdot \cos t\,dt = 3\cdot 3\int_0^{\pi/2}\cos^2 t\,dt =$ $\quad 1-\cos\alpha = 2\sin^2\frac{\alpha}{2}$, $\quad 1+\cos\alpha = 2\cos^2\frac{\alpha}{2}$

$= 9\int_0^{\pi/2}\frac{1+\cos 2t}{2}\,dt = 9\left(\frac{1}{2} t\Big|_0^{\pi/2} - \frac{1}{2}\int_0^{\pi/2}\cos 2t\,dt\right) =$

$= 9\left(\frac{\pi}{4} - \frac{1}{2}\cdot\frac{1}{2}\sin 2t\Big|_0^{\pi/2}\right) = \boxed{\frac{9\pi}{4}} - \frac{9}{4}(0)$

$P = \boxed{\frac{3\pi}{4} - 3}$

6) Data je kriva $y^2=6(2-x)$ i tačka $M(-4, y>0)$ koja pripada krivoj.

a) odrediti jnu tang. u tački M;

b) Odrediti P figure ograničene krivom i jnom tang. i u tački M i Ox-osom.

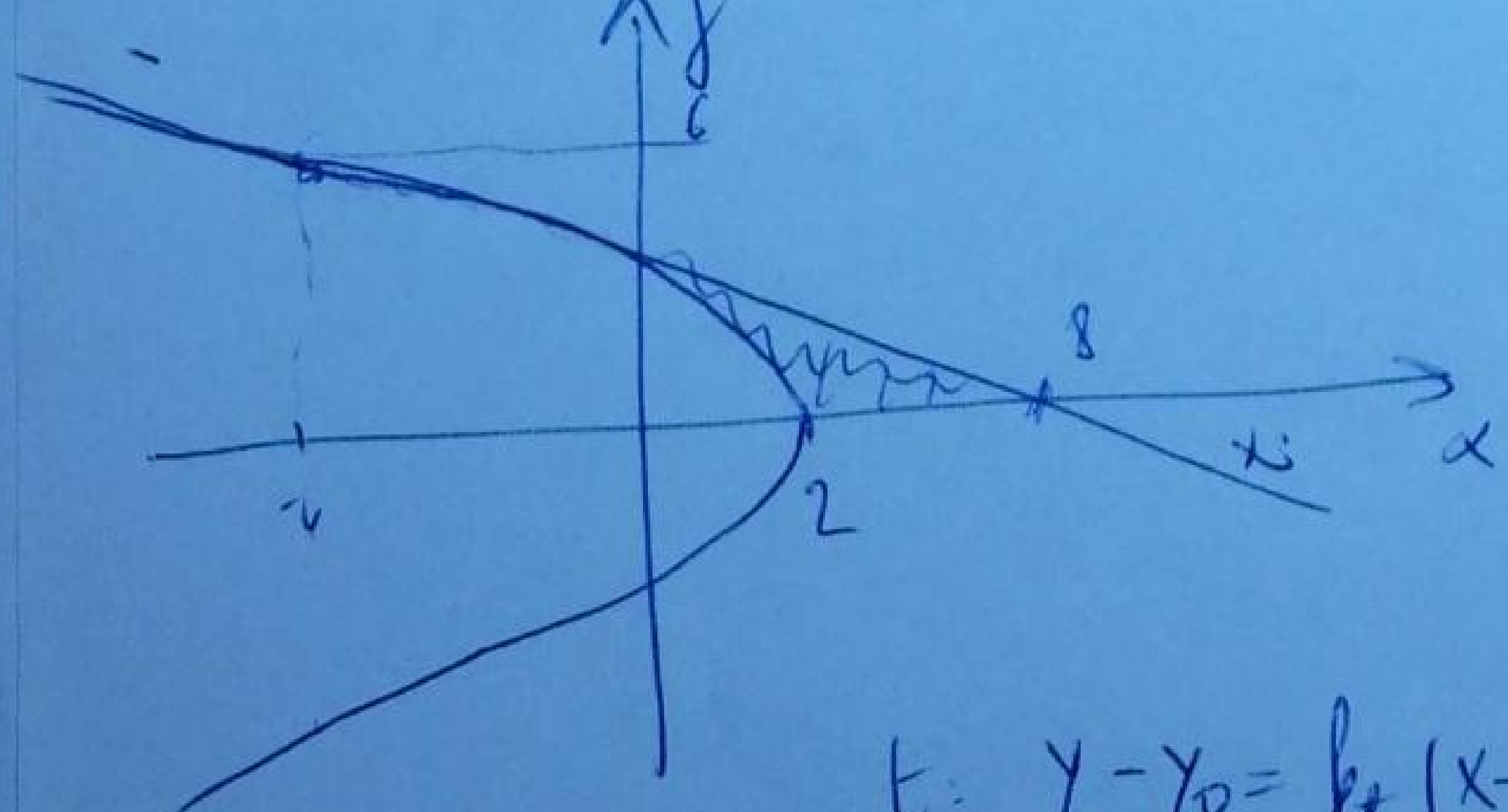
Rj: $M \in$ krivoj $\Rightarrow y^2 = 6(2+4)$ $\qquad M(-4,6)$

$\underline{y=\pm 6} \Rightarrow y>0 \Rightarrow \underline{y=6}$

$x=0 \Rightarrow y^2=12$

$y=\pm\sqrt{12}$

$y=0 \quad x=2$



$k_t = y'(M)$

$2yy' = -6$

$y' = \frac{-6}{2y}\Big|_{M(-4,6)} = -\frac{1}{2}$

t: $y-y_0 = k_t(x-x_0)$

$y-6 = -\frac{1}{2}(x+4)$

t: $\underline{y = -\frac{x}{2} + 4}$

$P = ?$ $\quad t: \; y = 0 \Rightarrow \frac{x}{2} = 4$

$\boxed{x = 8}$

$$P = \int_{-4}^{8} \left(4 - \frac{x}{2}\right) dx + \int_{-4}^{2} \sqrt{6(2-x)}\, dx = \qquad \begin{array}{l} 6(2-x) = t \\ -6\,dx = dt \end{array} \qquad \begin{array}{c|c|c|} x & -4 & 2 \\ \hline t & 36 & 0 \end{array}$$

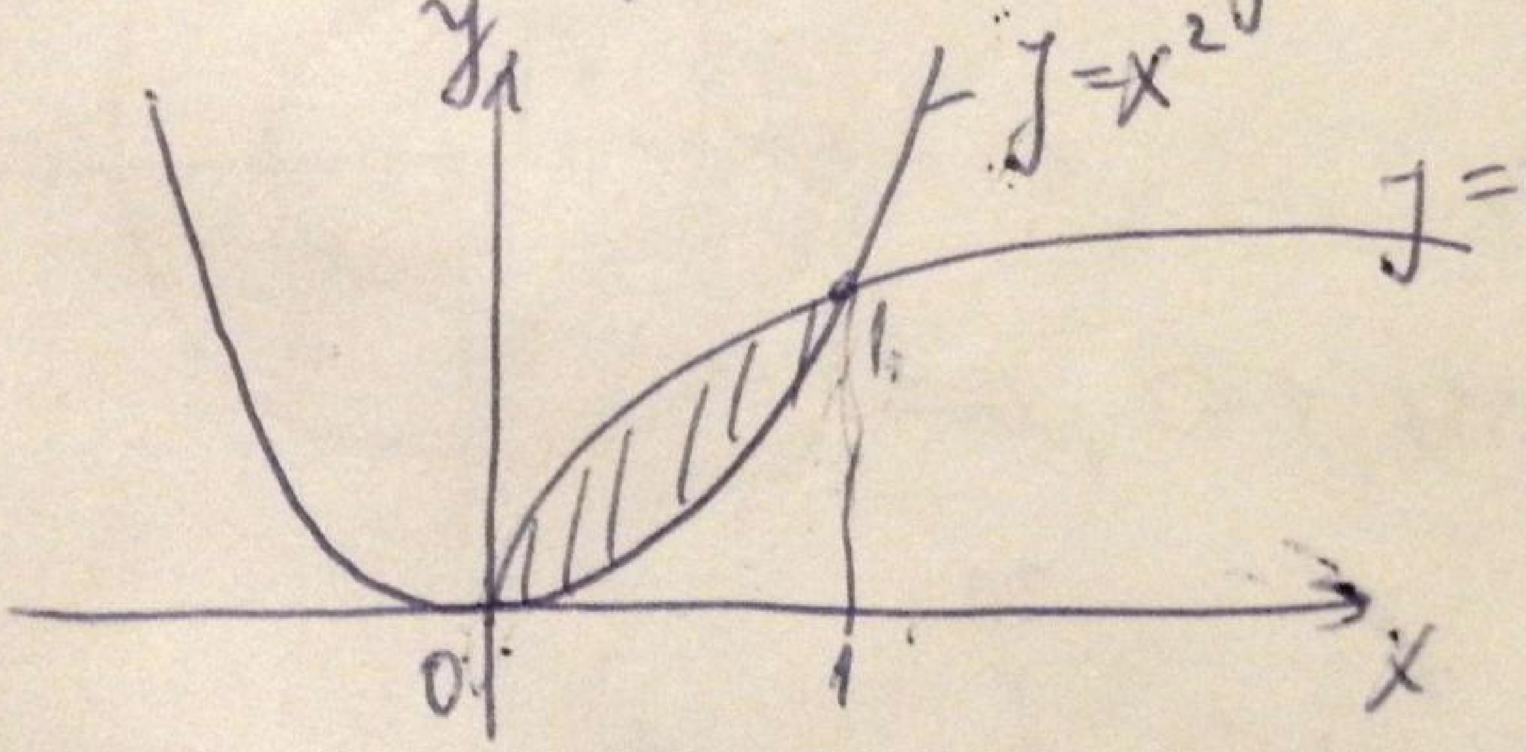
$$= 4x \Big|_{-4}^{8} - \frac{1}{2}\frac{x^2}{2}\Big|_{-4}^{8} + \frac{1}{6}\int_{36}^{0} \sqrt{t}\, dt =$$

$$= 4(8+4) - \frac{1}{4}(64 - 16) + \frac{1}{6}\cdot\frac{2t^{3/2}}{3}\Big|_{36}^{0} =$$

$$= 4 \cdot 32 - \frac{1}{4}48 + \frac{1}{9}\left(\sqrt{0} - \sqrt{36^3}\right) = \ldots$$

1) Izračunati površinu ograničenu linijama $y = x^2$ i $y = \sqrt{x}$.

Rj:



$$y = x^2$$
$$y = \sqrt{x}$$

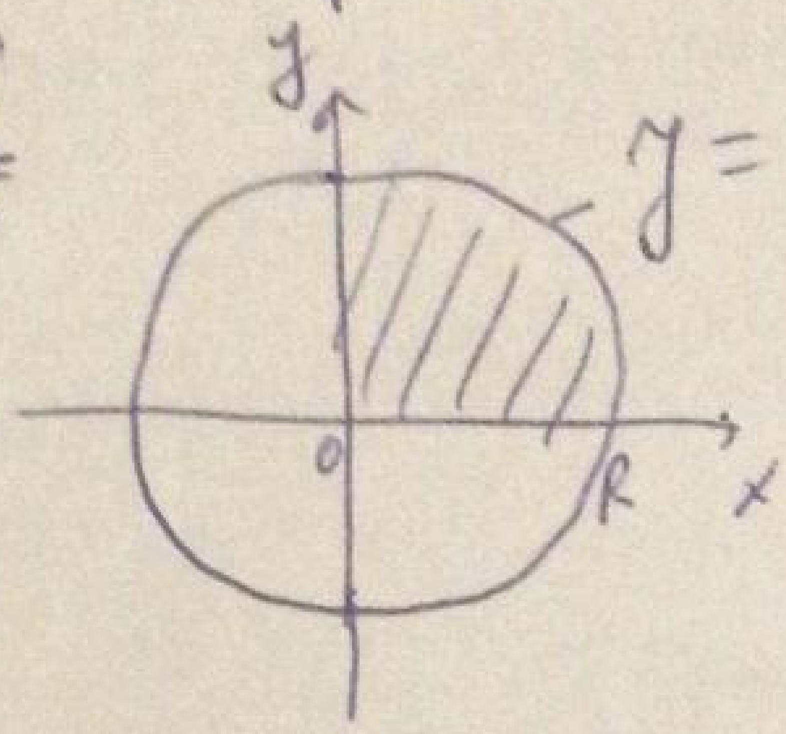
$$x^2 = \sqrt{x}$$
$$x^4 - x = 0$$
$$x(x^3 - 1) = 0$$
$$\underline{x = 0} \lor x^3 = 1$$
$$\underline{x = 1}$$

$$P = \int_0^1 (\sqrt{x} - x^2)\,dx = \int_0^1 \sqrt{x}\,dx - \int_0^1 x^2\,dx = \frac{x^{3/2} \cdot 2}{3}\Big|_0^1 - \frac{x^3}{3}\Big|_0^1 =$$

$$= \frac{2}{3}(1 - 0) - \frac{1}{3}(1 - 0) = \frac{2}{3} - \frac{1}{3} = \underline{\underline{\frac{1}{3}}}$$

2) Naći površinu kruga $x^2+y^2=R^2$.

5) Rj.



$$y=\pm\sqrt{R^2-x^2}$$

$$P=4\int_0^R\sqrt{R^2-x^2}\,dx=$$

$= \ulcorner\ x=R\sin t \Rightarrow dx=R\cos t$

x	0	R
t	0	$\frac{\pi}{2}$ $\lrcorner$

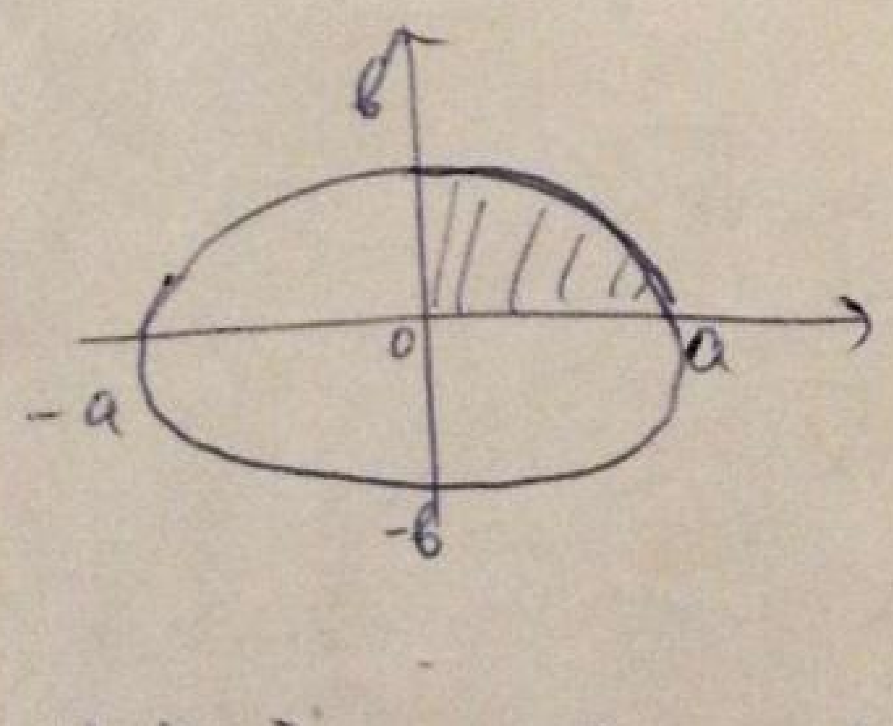
$$=4\int_0^{\pi/2}\sqrt{R^2-R^2\sin^2 t}\;R\cos t\,dt=$$

$$=4\int_0^{\pi/2}R^2\cos^2 t\,dt=4R^2\int_0^{\pi/2}\frac{1+\cos 2t}{2}\,dt=2R^2\,t\Big|_0^{\pi/2}+2R^2\cdot\frac{1}{2}\sin 2t\Big|_0^{\pi/2}$$

$$=2R^2\left(\frac{\pi}{2}-0\right)+R^2(\sin\pi-\sin 0)=\pi R^2+R^2(0-0)=\underline{\underline{R^2\pi}}$$

3) Izračunati P elipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

6) Rj. $\frac{y^2}{b^2}=1-\frac{x^2}{a^2}\Rightarrow y^2=b^2\left(1-\frac{x^2}{a^2}\right)\Rightarrow y=\pm\frac{b}{a}\sqrt{(a^2-x^2)}$

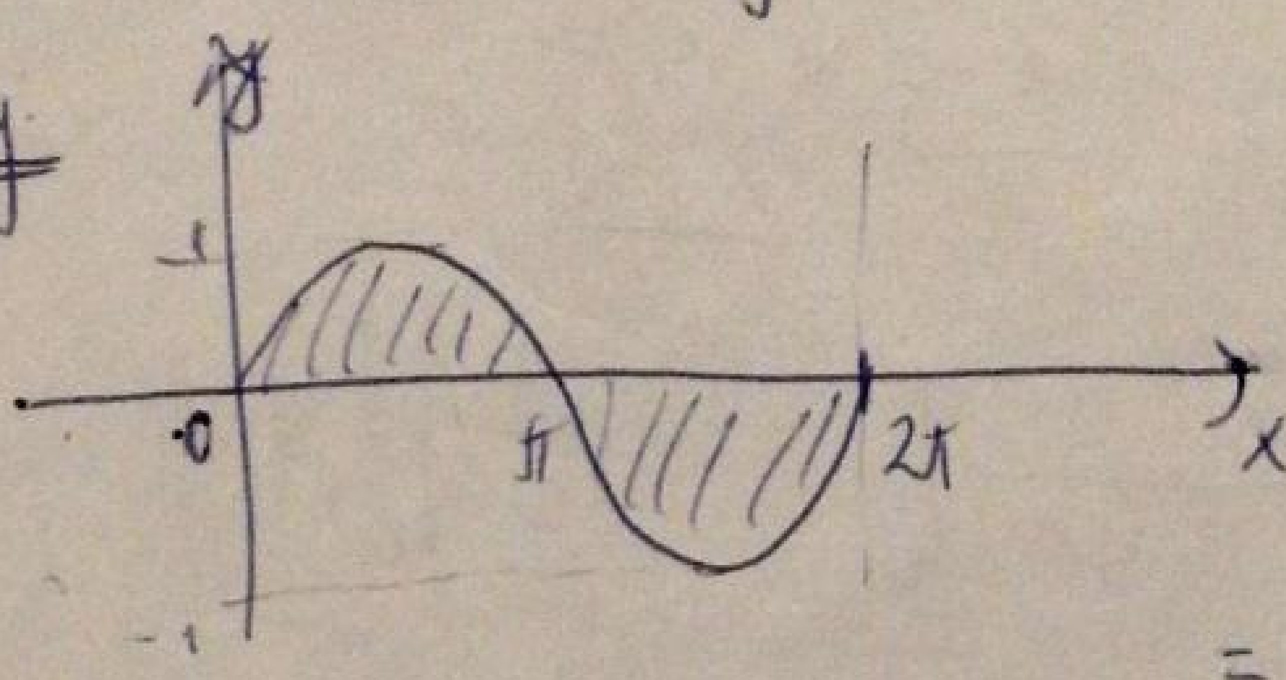


$$P=4\int_0^a\frac{b}{a}\sqrt{a^2-x^2}\,dx=4\frac{b}{a}\int_0^a\sqrt{a^2-x^2}\,dx=\ulcorner x=a\sin t\lrcorner$$

$$=\underline{\underline{ab\pi}}$$

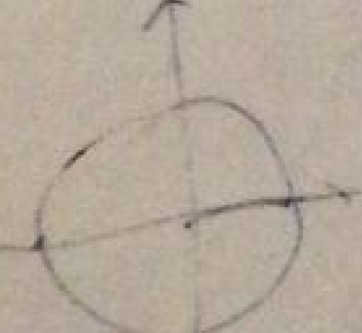
4) Naći površinu ograničenu krivom $y=\sin x$, $x=0$, $x=2\pi$.

Rj.



$$P=\int_0^{\pi}\sin x\,dx-\int_{\pi}^{2\pi}\sin x\,dx=$$

$$=-\cos x\Big|_0^{\pi}+\cos x\Big|_{\pi}^{2\pi}=-(\cos\pi-\cos 0)+\cos 2\pi-\cos\pi=$$

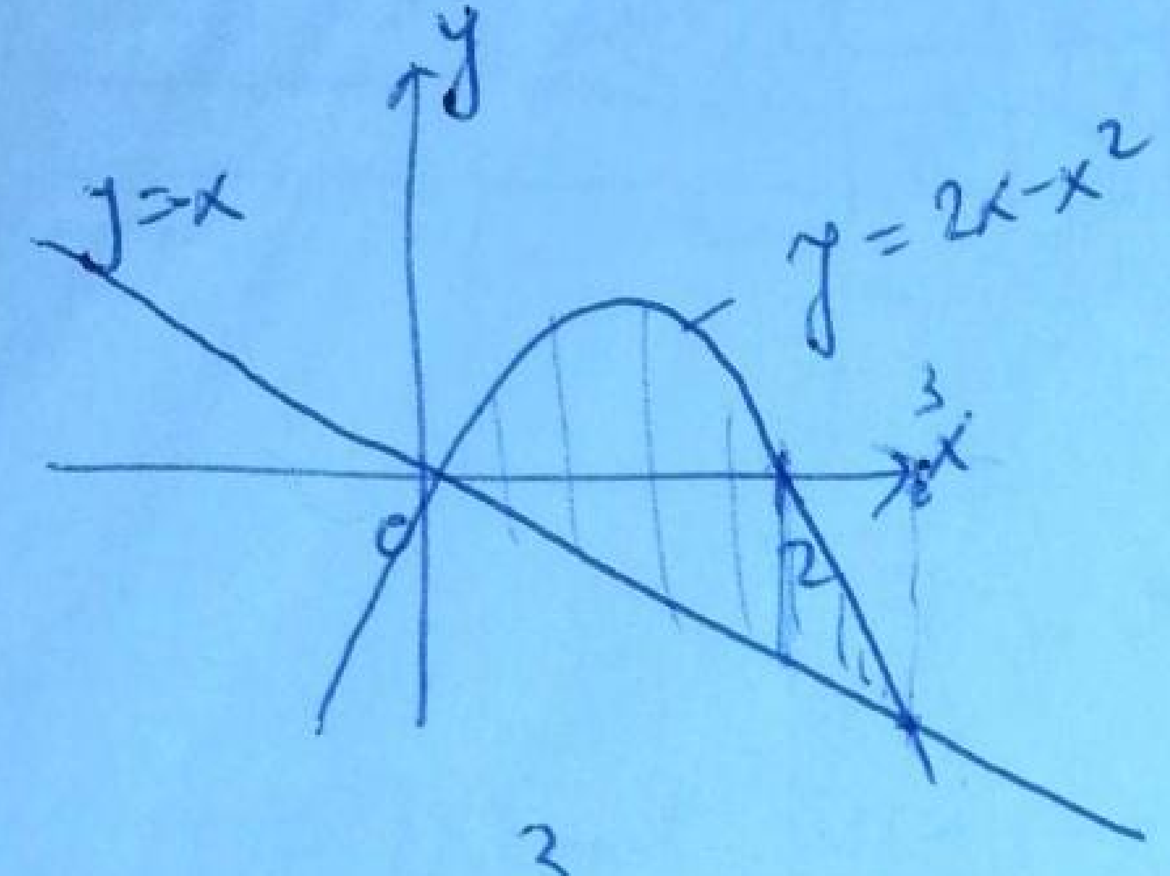


$$=-(-1-1)+1+1=\underline{\underline{4}}$$

5) Naći P ograničenu parabolom $y=2x-x^2$ i pravom $y=-x$. (3)

Rj.

$y=2x-x^2=x(2-x) \Rightarrow x=0 \vee x=2$



$2x-x^2=-x$

$3x-x^2=0$

$x(3-x)=0$

$\underline{x=0} \vee \underline{x=3}$

$$\Rightarrow P=\int_0^3\left((2x-x^2)-(-x)\right)dx=\int_0^3(3x-x^2)\,dx=3\,\frac{x^2}{2}\Big|_0^3-\frac{x^3}{3}\Big|_0^3=$$

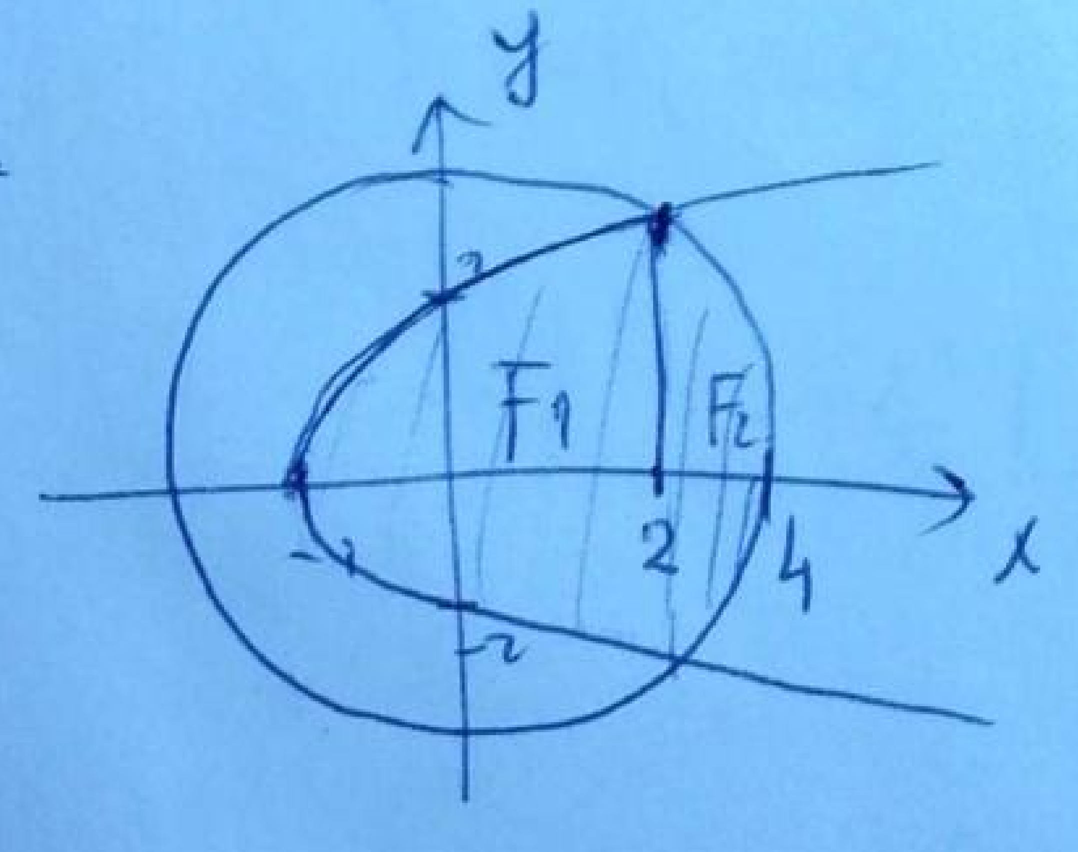
$$=3\left(\frac{9}{2}-0\right)-\left(\frac{27}{3}-0\right)=\frac{27}{2}-\frac{27}{3}=\frac{3\cdot27-27\cdot2}{6}=\frac{27(3-2)}{6}=$$

$$=\frac{27}{6}$$

6) Izračunati P ograničenu krivim:

$$\begin{cases} x^2+y^2=16 \\ y^2=4(x+1) \end{cases}$$

Rj.



$y^2=4(x+1)$

$x=0 \to y=\pm 2$

$y=0 \Rightarrow x=-1$

$x^2+4x+4=16$

$x^2+4x+12=0$

$\boxed{x_1=2},\ x_2=-6$

$P=P(F_1)+P(F_2)$

$$P(F_1)=\int_{-1}^{2}2\sqrt{x+1}\,dx-\int_{-1}^{2}-2\sqrt{x+1}\,dx=2\int_{-1}^{2}\sqrt{x+1}\,dx+2\int_{-1}^{2}\sqrt{x+1}\,dx=$$

$$=4\int_{-1}^{2}\sqrt{x+1}\,dx=\begin{bmatrix} x+1=t & \begin{array}{c|c|c} x & -1 & 2 \\ \hline t & 0 & 3 \end{array} \\ dx=dt & \end{bmatrix}=4\int_0^3\sqrt{t}\,dt=$$

$$= 4\int_0^3 t^{1/2}\,dt = 4\cdot\frac{2t^{3/2}}{3}\Big|_0^3 = \frac{8}{3}\sqrt{27} = \underline{\underline{8\sqrt{3}}}$$

$$P(F_2) = 2\int_2^4 \sqrt{16-x^2}\,dx = \begin{bmatrix} x = 4\sin t \\ dx = 4\cos t\,dt \end{bmatrix} \quad \begin{array}{c|c|c} x & 2 & 4 \\ \hline t & \pi/6 & \pi/2 \end{array} =$$

$$t = \arcsin\frac{x}{4}$$

$$= 2\int_{\pi/6}^{\pi/2} 4\sqrt{16-16\sin^2 t}\,\cos t\,dt = 8\cdot 4\int_{\pi/6}^{\pi/2}\cos^2 t\,dt =$$

$$= 32\int_{\pi/6}^{\pi/4}\frac{1+\cos 2t}{2}\,dt = 16\,t\Big|_{\pi/6}^{\pi/2} + 16\,\frac{1}{2}\sin 2t\Big|_{\pi/6}^{\pi/2} =$$

$$= 16\left(\frac{\pi}{2}-\frac{\pi}{6}\right) + 8\left(\sin\pi - \sin\frac{\pi}{3}\right) = \frac{16\pi}{3} + 8\left(0-\frac{\sqrt{3}}{2}\right) =$$

$$= \frac{16\pi}{3} - 4\sqrt{3}$$

$$P = 8\sqrt{3} + \frac{16\pi}{3} - 4\sqrt{3} = \boxed{\frac{16\pi}{3} + 4\sqrt{3}}$$

7) $\begin{cases} x^2+y^2=2 \\ y = x^2 \end{cases}$

$x^2 + x^4 = 2$

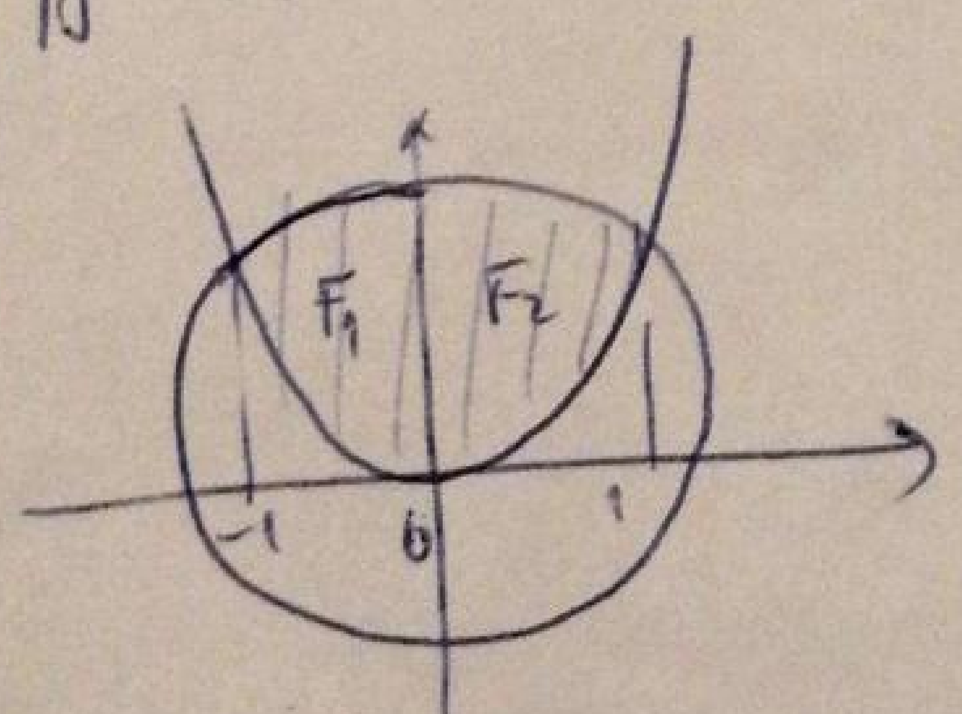
$x^2 = t$

$t^2 + t = 2$

$t_1 = 1,\ t_2 = -2\ (\perp)$

$x = \pm 1$

~~Rj~~



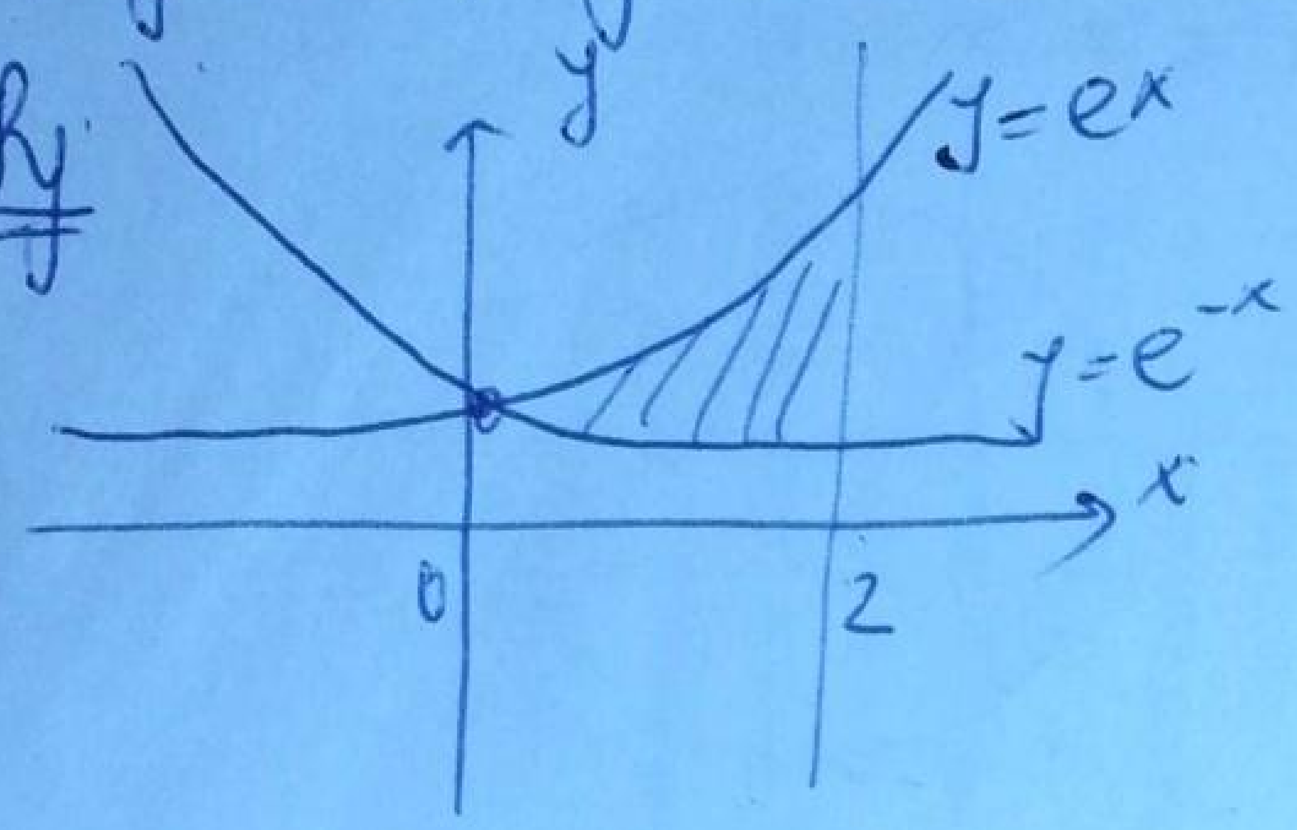
$$P(F_1) = \int_{-1}^{0}\left(\sqrt{2-x^2} - x^2\right)dx = \ldots$$

$$P(F_2) = \int_0^1\left(\sqrt{2-x^2} - x^2\right)dx = \ldots$$

$| =-$

8) $y=e^{x},\ y=e^{-x},\ x=2$

Rj



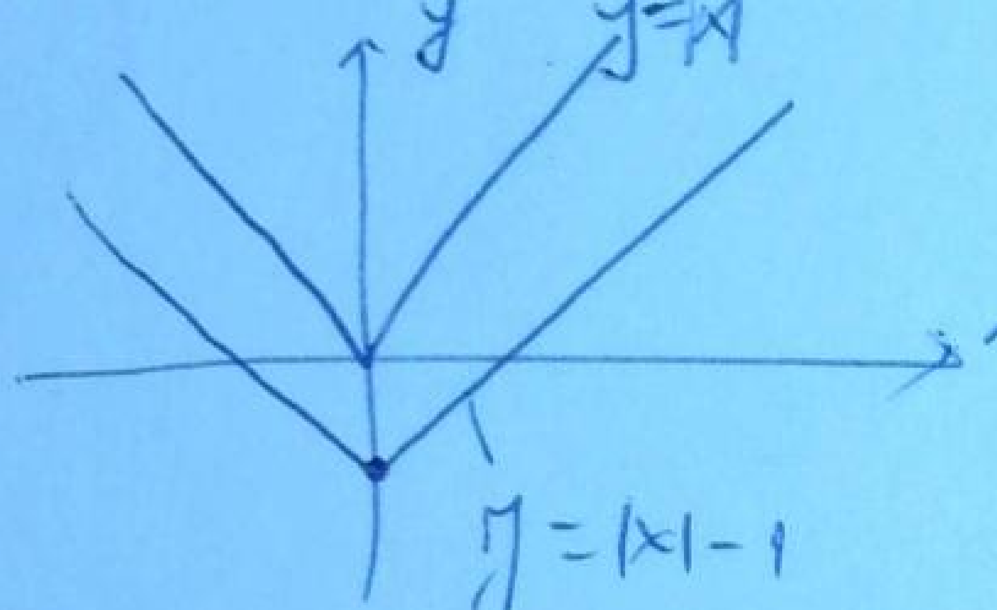
$$P=\int_0^2 (e^{x}-e^{-x})\,dx = e^{x}\Big|_0^2 + \frac{1}{e^{x}}\Big|_0^2 =$$

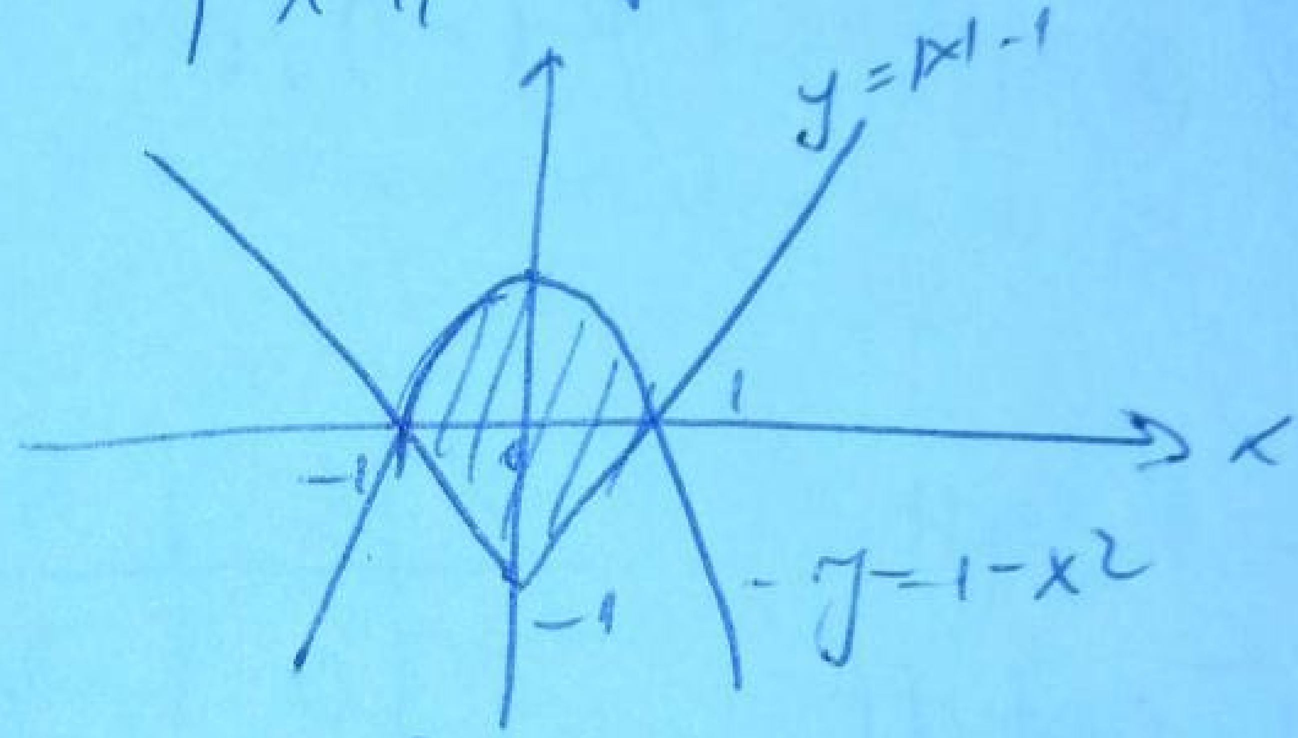
$$= e^{2}-1+\frac{1}{e^{2}}-1=\frac{e^{4}+1}{e^{2}}-2.$$

9) $y=|x|-1,\ y=1-x^{2}.$

$$y=\begin{cases} x-1, & x\geq 0\\ -x-1, & x<0\end{cases}$$

Rj





$|x|-1=1-x^{2}$

$|x|+x^{2}=2 \qquad \cancel{x+1} \qquad x^{2}+x-2=0$

$$P(F_1)=\int_{-1}^{0}\left((1-x^{2})-(-x-1)\right)dx = \int_{-1}^{0}(1-x^{2}+x+1)\,dx=$$

$$|x|-1=\begin{cases} x-1, & x\geq 0\\ -x-1, & x<0\end{cases} \qquad = \int_{-1}^{0}-x^{2}\,dx+\int_{-1}^{0}x\,dx+2x\Big|_{-1}^{0}=$$

$$=-\frac{x^{3}}{3}\Big|_{-1}^{0}+\frac{x^{2}}{2}\Big|_{-1}^{0}+2(0+1)=-\left(0+\frac{1}{3}\right)-\frac{1}{2}+2=$$

$$=\frac{7}{6}$$

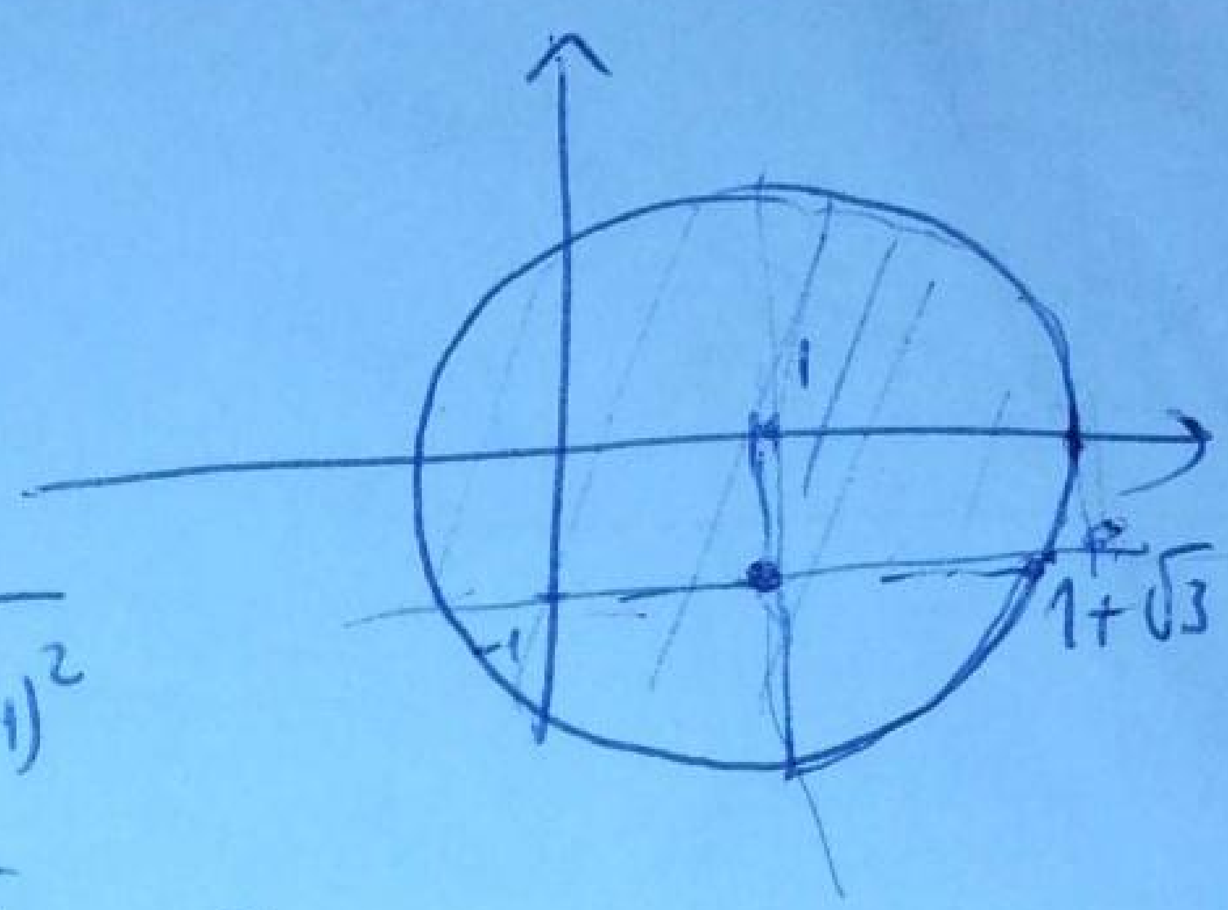
$$P(F_2)=\int_0^1\left(\overset{1-x^{2}-x+1}{1-x^{2}}-(x-1)\right)dx=\int_0^1(-x^{2}-x+2)\,dx=-\frac{x^{3}}{3}\Big|_0^1-\frac{x^{2}}{2}\Big|_0^1+2x\Big|_0^1=$$

$$=\frac{7}{6}$$

$$\Rightarrow P=\frac{14}{6}=\frac{7}{3}$$

10) P figure ograničene sa:

$x^2+y^2-2x+2y-1=0.$



Rj:

$(x-1)^2+(y+1)^2-1-1=1$

$(x-1)^2+(y+1)^2=3 \Rightarrow y=-1\pm\sqrt{3-(x-1)^2}$

kružnica sa centrom u $(1,-1)$ i $r=\sqrt{3}$

$$P=4\int_1^{1+\sqrt{3}}\left(\left(\sqrt{3-(x-1)^2}-1\right)-(-1)\right)dx=4\int_1^{1+\sqrt{3}}\sqrt{3-(x-1)^2}\,dx=$$

$= \left[\begin{array}{l} x-1=\sqrt{3}\sin t \\ dx=\sqrt{3}\cos t\,dt \end{array}\right.$

x	1	$1+\sqrt{3}$
t	0	$\pi/2$

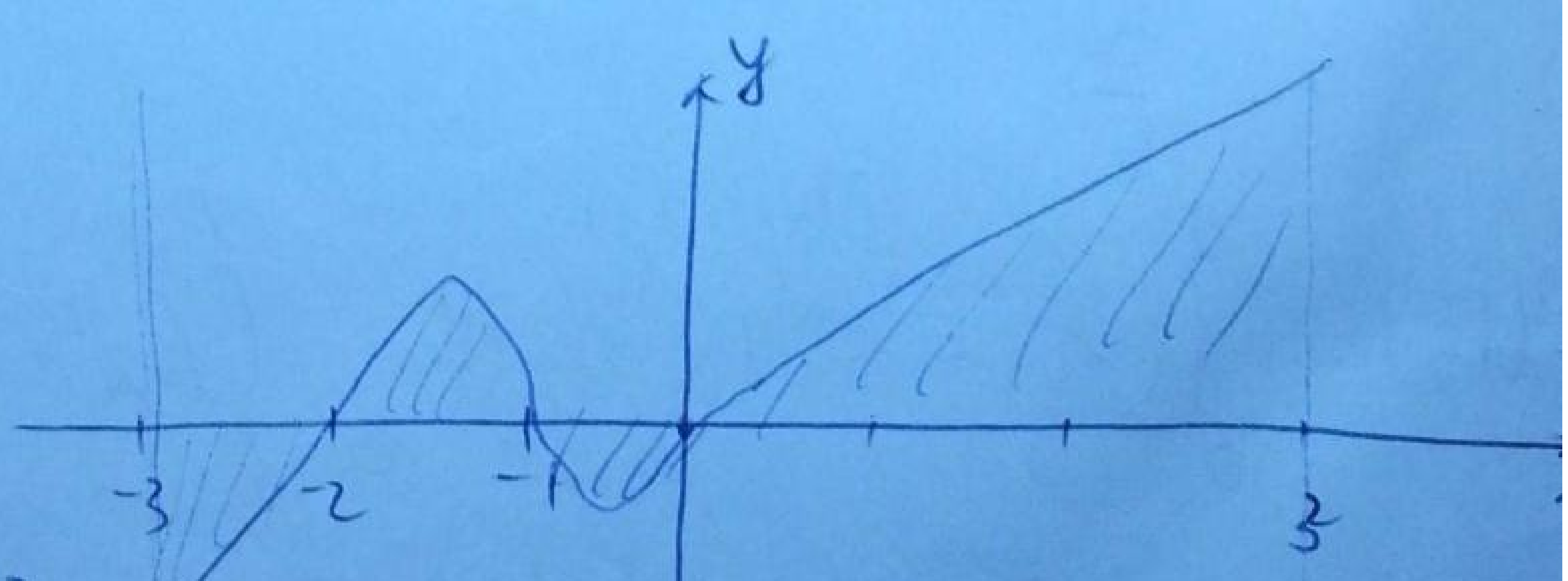
$=$

$t=\arcsin\frac{x-1}{\sqrt{3}}$

$$=4\sqrt{3}\sqrt{3}\int_0^{\pi/2}\cos^2 t\,dt=4\sqrt{3}\int_0^{\pi/2}\frac{1+\cos 2t}{2}\,dt=4\sqrt{3}\sqrt{3}\left(\frac{1}{2}\frac{\pi}{2}+\frac{1}{2}(\sin\pi-\sin 0)\right)$$

$$=\overset{12}{\cancel{4\sqrt{3}\sqrt{3}}}\,\frac{1}{2}\left(\frac{\pi}{2}\right)=\frac{6\pi}{2}=\underline{\underline{3\pi}}$$

11) Izračunati P ograničene lukom krive $y=x^3+3x^2+2x$, x osom i pravima $x=3$ i $x=-3$.

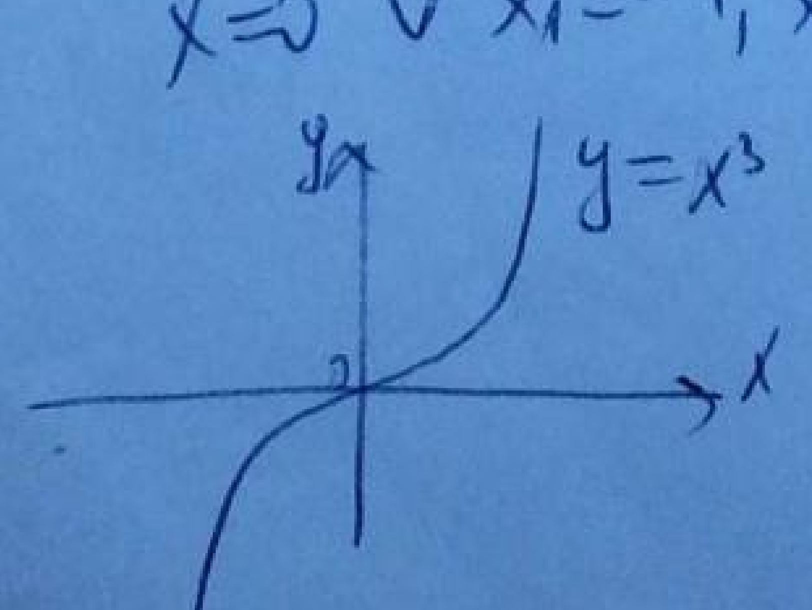


Rj:

$y=x^3+3x^2+2x$

$y=0 \Rightarrow x(x^2+3x+2)=0$

$x=0 \vee x_1=-1,\ x_2=-2$



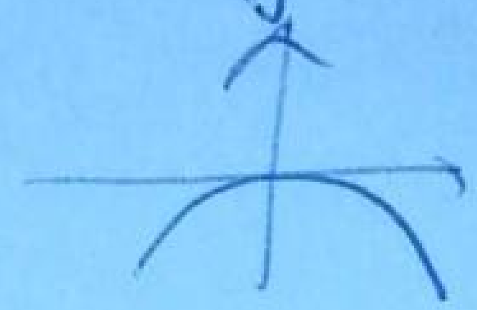
$$P=\int_{-3}^{-2}-y\,dx+\int_{-2}^{1}y\,dx+\int_{-1}^{0}-y\,dx+$$

$$+\int_0^3 y\,dx=$$

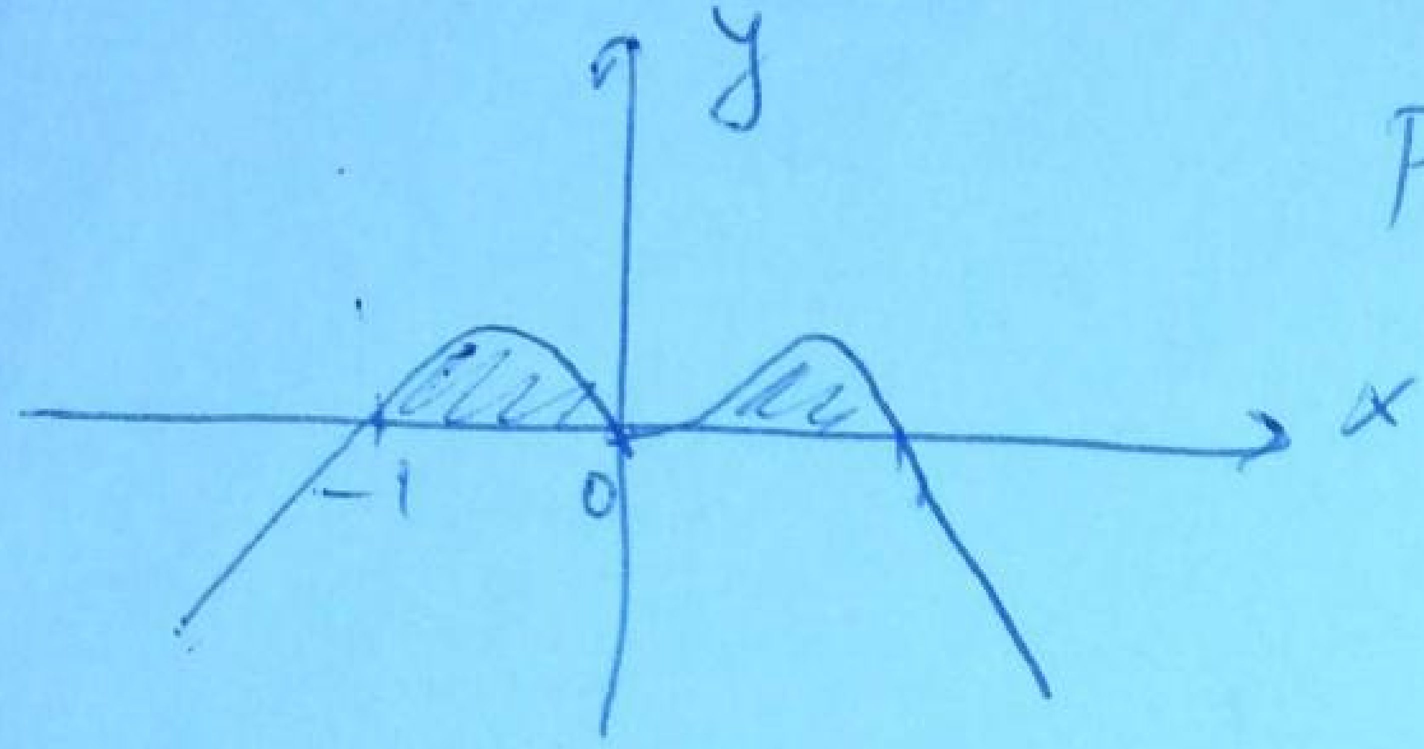
11) Izračunati P ograničenu krivom $y=x^2(1-x^2)$ i x-osom. ⑤

Rj:

$y=x^2(1-x^2) \Rightarrow y \approx -x^4$



$y=0 \Rightarrow \quad x=0 \lor x=\pm 1$



$P=\int_{-1}^{0} y\,dx+\int_{0}^{1} y\,dx=, \quad y=x^2(1-x^2)$

...

13) Naći P ograničenu lukom krive $x=6-y-y^2$ i y-osom.

Rj:

$x=6-(y+y^2)$

$x=6-(y+\frac{1}{2})^2+\frac{1}{4}$

$x=\frac{25}{4}-(y+\frac{1}{2})^2$

$x-\frac{25}{4}=-(y+\frac{1}{2})^2$

$\frac{25}{4}-x=(y+\frac{1}{2})^2$

$-(x-\frac{25}{4})=(y+\frac{1}{2})^2$

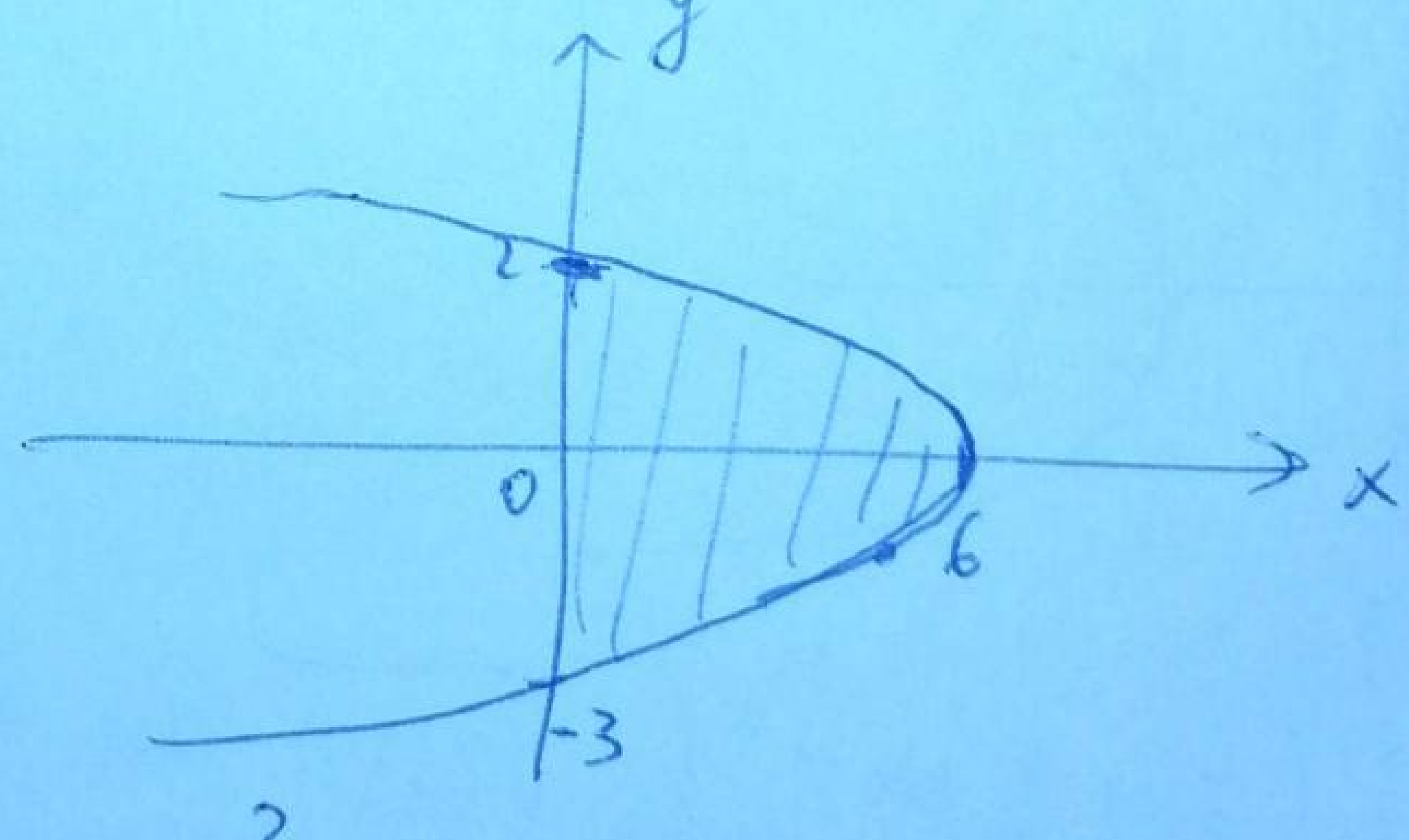
$\Downarrow$

parabola $O'(\frac{25}{4},-\frac{1}{2})$-tjeme

$y=0 \Leftrightarrow x=6$

$x=0 \Rightarrow \quad 6-y+y^2=0$

$y_1=2, \; y_2=-3$



$P=\int_{-3}^{2}\underbrace{(6-y-y^2)}_{x(y)}dy=6y\Big|_{-3}^{2}-\frac{y^2}{2}\Big|_{-3}^{2}-\frac{y^3}{3}\Big|_{-3}^{2}$

$= \ldots$

14) U tački $P(3,y_0)$, $y_0>0$ parabole $y^2=2(x-1)$ povučena je tangenta. Izračunati P figure ograničene tom tangentom, parabolom i x-osom.

Rj:

$y^2 = 2(x-1) \Rightarrow y = \pm\sqrt{2(x-1)}$

$y=0 \Leftrightarrow x=1$

$x=0 \Leftrightarrow y^2 = (-2) \Rightarrow$ nema pr-sa y-osom.

$P(3, y_0) \in$ parabole $\Rightarrow y_0^2 = 2(3-1) \Rightarrow y_0^2 = 4 \Rightarrow \underline{y_0 = \pm 2}$

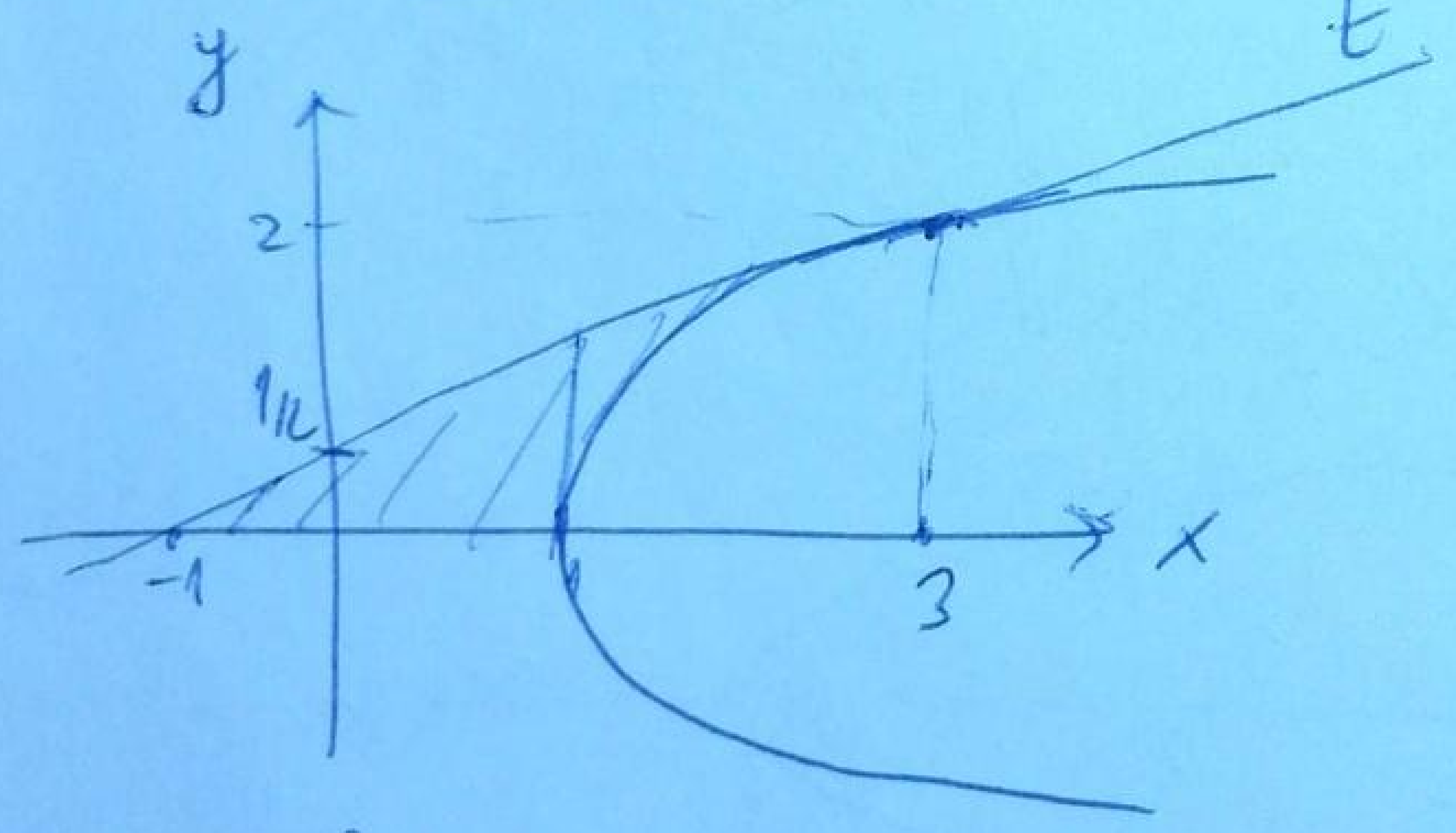
$y_0 > 0 \Rightarrow \underline{P(3,2)}$

$t: \ y-2 = k(x-3), \ k = y'(3,2)$

$y^2 = 2(x-1) \Rightarrow 2yy' = 2 \Rightarrow y' = \frac{1}{y} = \frac{1}{2}$

$\Rightarrow t: \ y-2 = \frac{1}{2}(x-3)$

$$\boxed{y = \frac{1}{2}x + \frac{1}{2}}$$



t: za $x=0 \Rightarrow y = \frac{1}{2}$

$y=0 \Rightarrow x=-1$

$$P = \int_{-1}^{3}\left(\frac{1}{2}x+\frac{1}{2}\right)dx - \int_{1}^{3}\sqrt{2(x-1)}\,dx = \frac{1}{2}\frac{x^2}{2}\Big|_{-1}^{3} + \frac{1}{2}x\Big|_{-1}^{3} -$$

$$-\sqrt{2}\int_{1}^{3}\underbrace{\sqrt{x-1}}_{x-1=t}\,dx = \ldots$$

$\underline{\text{II način}}$: $P = \int_{-1}^{1}\left(\frac{1}{2}x+\frac{1}{2}\right)dx + \int_{1}^{3}\left(\frac{1}{2}x+\frac{1}{2} - \sqrt{2(x-1)}\right)dx = .$

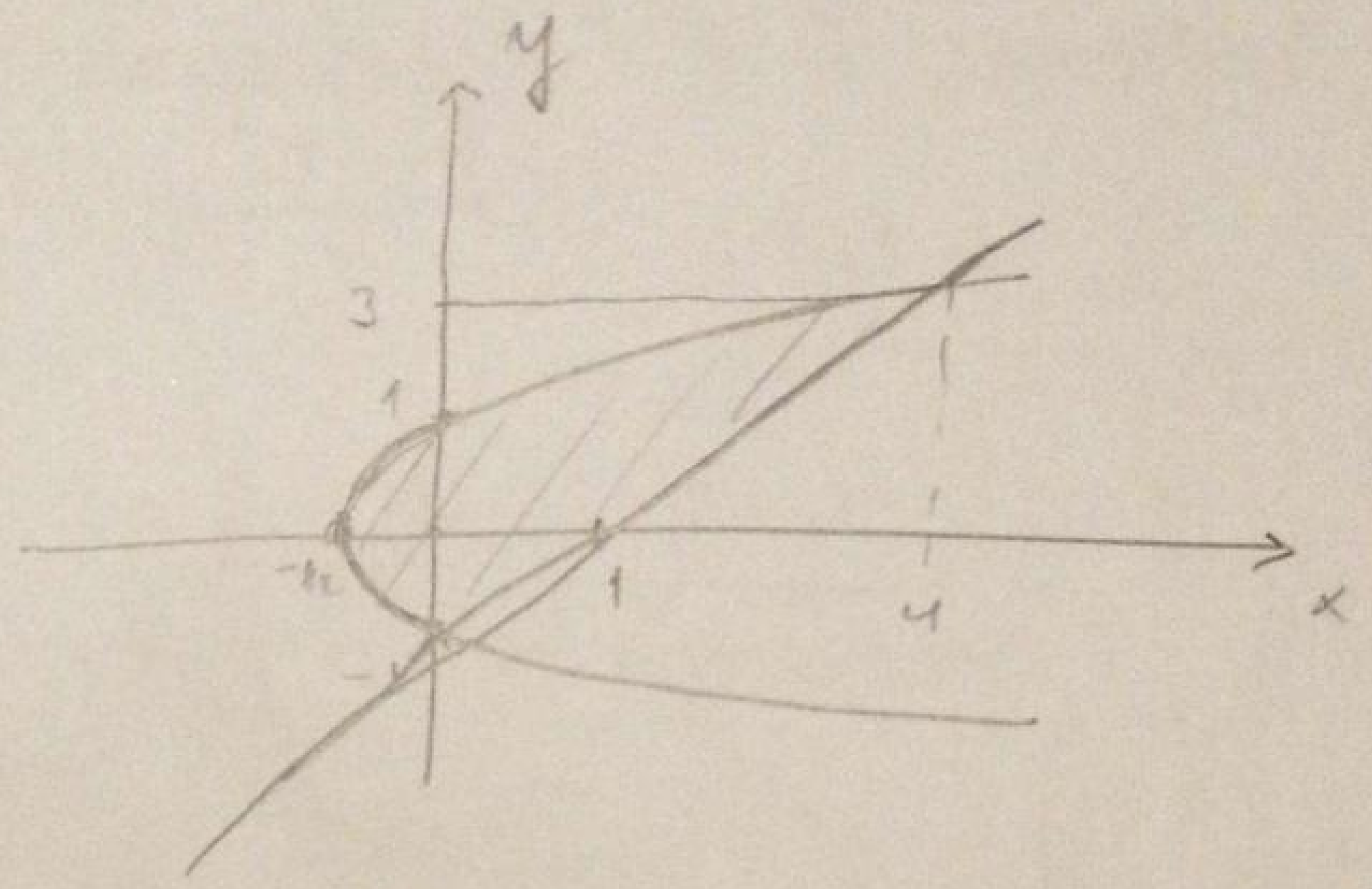
1) Izračunaj površinu ograničenu krivom $y^2 = 2x+1$ i pravom $y = x-1$.

①

$y^2 = 2x+1$

$x=0 \Rightarrow y^2 = +1 \Leftrightarrow y = \pm 1$

$y=0 \Rightarrow x = -\frac{1}{2}$



$y^2 = 2x+1 \Rightarrow y = \pm\sqrt{2x+1}$

$y = x-1, \quad x=0 \Rightarrow y = -1$
$\qquad\qquad\quad y=0 \Rightarrow x = 1$

Presjek prave i krive: $\begin{cases} y = x-1 \rightarrow x = y+1 \\ y^2 = 2x+1 \end{cases}$

$y^2 = 2(y+1)+1$

$y^2 - 2y - 3 = 0$

$y_{1,2} = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2}$

$y_1 = 3, \quad y_2 = -1 \qquad y = 3 \Rightarrow x = 4$

I način

$$P = \int_{-1}^{3} \left((y+1) - \frac{y^2-1}{2}\right) dy = \ldots = \frac{16}{3}$$

II način

$$P = 2\int_{-1/2}^{0} \sqrt{2x+1}\, dx + \int_{0}^{4} \left(\sqrt{2x+1} - (x-1)\right) dx =$$

2) U presječnim tačkama prave $x - y + 1 = 0$ i parabole $y = x^2 - 4x + 5$ povučene su tangente na parabolu. Izračunati P ograničenu parabolom i tangentama

R_j

$$\begin{cases} y = x^2-4x+5 \\ x-y+1=0 \end{cases}$$

$y = x+1$

$x^2-4x+5-x-1=0$

$x^2-5x+4=0$

$x_{1,2} = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5\pm 3}{2}$

$\underline{x_1 = 4}$ $\quad$ $\underline{x_2 = 1}$

$x^2-4x+5=0$

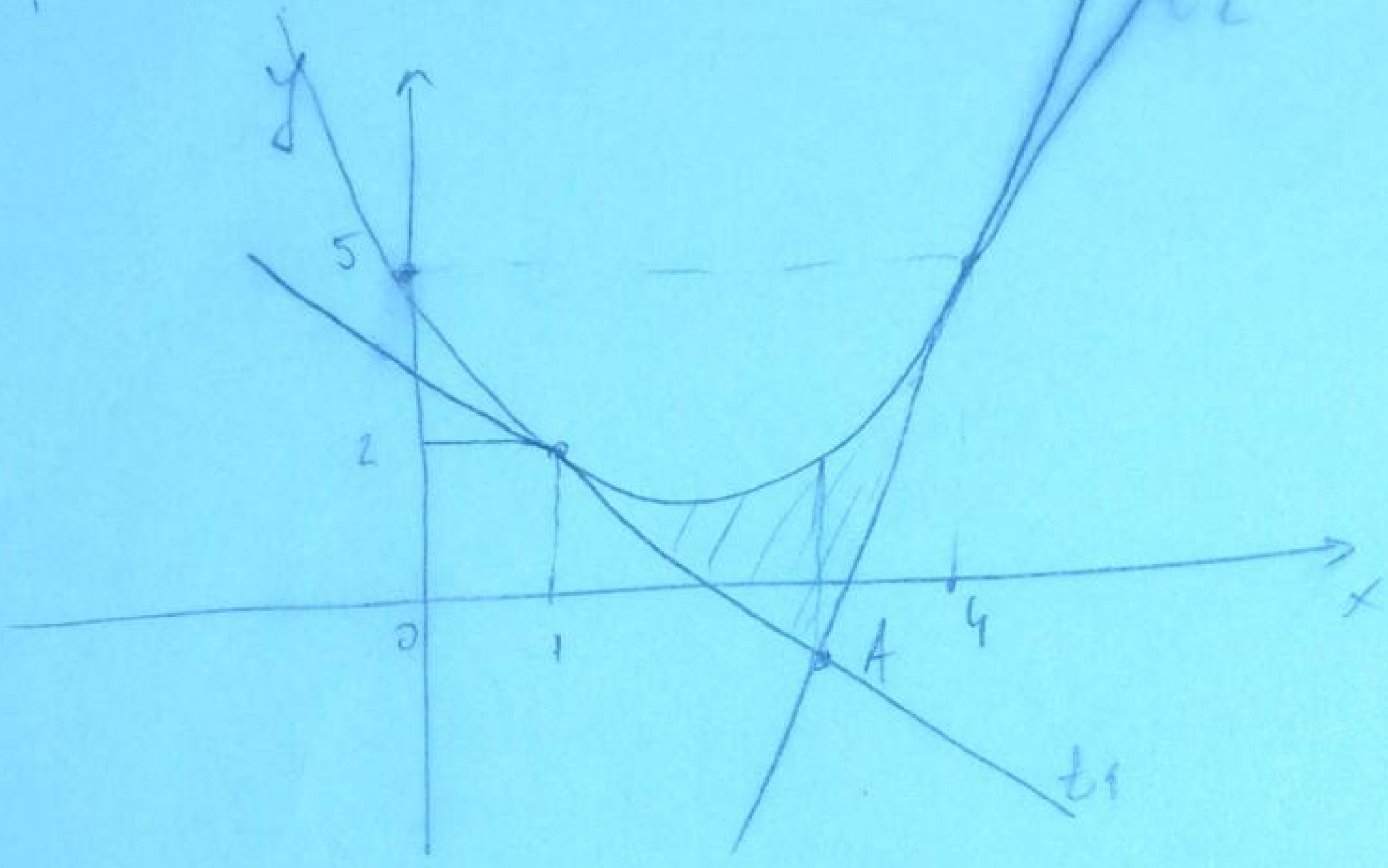
$x_{1,2} = \frac{4 \pm \sqrt{16-20}}{2} \notin \mathbb{R}$

$x=0 \Rightarrow y=5$

$x=1 \Rightarrow y=2$

$x=4 \Rightarrow y=5$

$y' = 2x-4$



$t_1 \quad y-y_0 = k_t(x-x_0)$

$y-2 = -2(x-1)$

$\underline{y = -2x+4}$

$t_2: \quad y-5 = 4(x-4)$

$\underline{y = 4x-11}$

$$P = \int_1^{5/2} \left[(x^2-4x+5)-(-2x+4)\right]dx + \int_{5/2}^{4} \left[(x^2-4x+5)-(4x-11)\right]dx$$

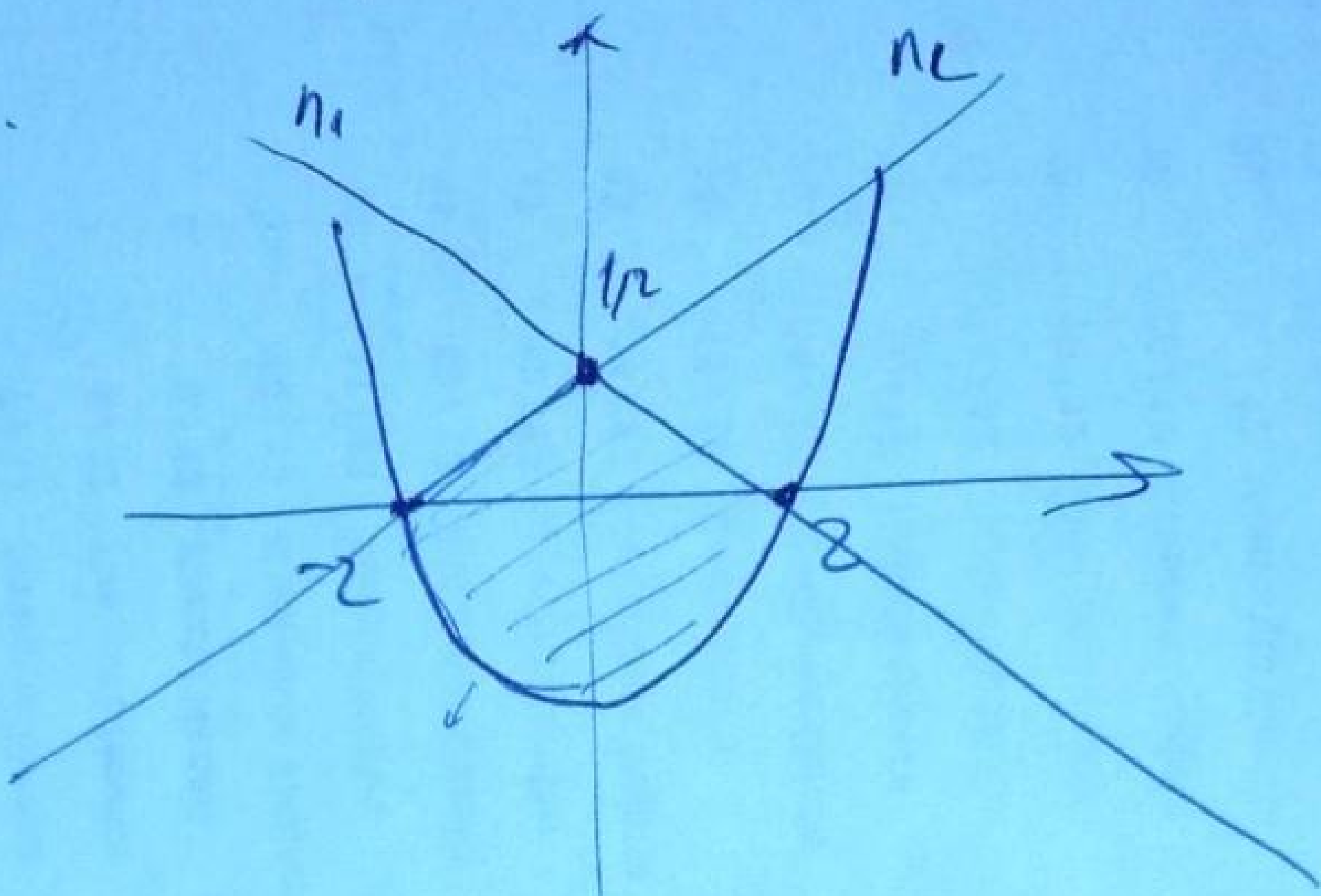
$t_1 \cap t_2 = \{A\}$

$-2x+4 = 4x-11$

$-6x = -15$

$\boxed{x = \frac{5}{2}}$

(1) U tačkama presjeka prave $y=0$ i parabole $y=x^2-4$ povučene su normale na parabolu. Naći površinu ograničenu parabolom i pravima.



Rj:

$$0=x^2-4$$
$$x^2=4$$
$$\underline{x=\pm 2}$$

n_1: $y-y_0=k_n(x-x_0)$ $\qquad (x_0,y_0)=(-2,0)$

$k_n=-\frac{1}{k_t}$, $\qquad (x_1,y_1)=(2,0)$

$$y-0=k_n(x+2)$$

$$k_n=-\frac{1}{k_t},\quad k_t=y'(A)\Rightarrow k_n=\frac{1}{4}$$

$$y=x^2-4\Rightarrow y'=2x=2\cdot(-2)=-4$$

$$y-0=\frac{1}{4}(x+2)$$

$$\boxed{y=\frac{1}{4}x+\frac{1}{2}}$$

$$y-0=k_n(x-2)$$

$$k_n=-\frac{1}{k_t}$$

$$k_t=4\Rightarrow k_n=-\frac{1}{4}$$

$$y=-\frac{1}{4}(x-2)$$

$$\boxed{y=-\frac{1}{4}x+\frac{1}{2}}$$

$$\frac{1}{4}x+\frac{1}{2}=-\frac{1}{4}x+\frac{1}{2}$$

$$\frac{2}{4}x=0$$

$$\boxed{x=0}\qquad \boxed{y=\frac{1}{2}}$$

$$P=2\int_{-2}^{0}\left(\left(\frac{1}{4}x+\frac{1}{2}\right)-(x^2-4)\right)dx=2\left[\int_{-2}^{0}\frac{1}{4}x\,dx+\frac{1}{2}\int_{-2}^{0}dx-\int_{-2}^{0}x^2dx+4\int_{-2}^{0}dx\right]=$$

$$=2\left[\frac{1}{4}\frac{x^2}{2}\Big|_{-2}^{0}+\frac{1}{2}x\Big|_{-2}^{0}-\frac{x^3}{3}\Big|_{-2}^{0}+4x\Big|_{-2}^{0}\right]=$$

$$=2\left[\frac{1}{4}\cdot\frac{4}{2}+1+\frac{8}{3}+8\right]=2\left[\frac{1}{2}+9+\frac{8}{3}\right]=\ldots$$